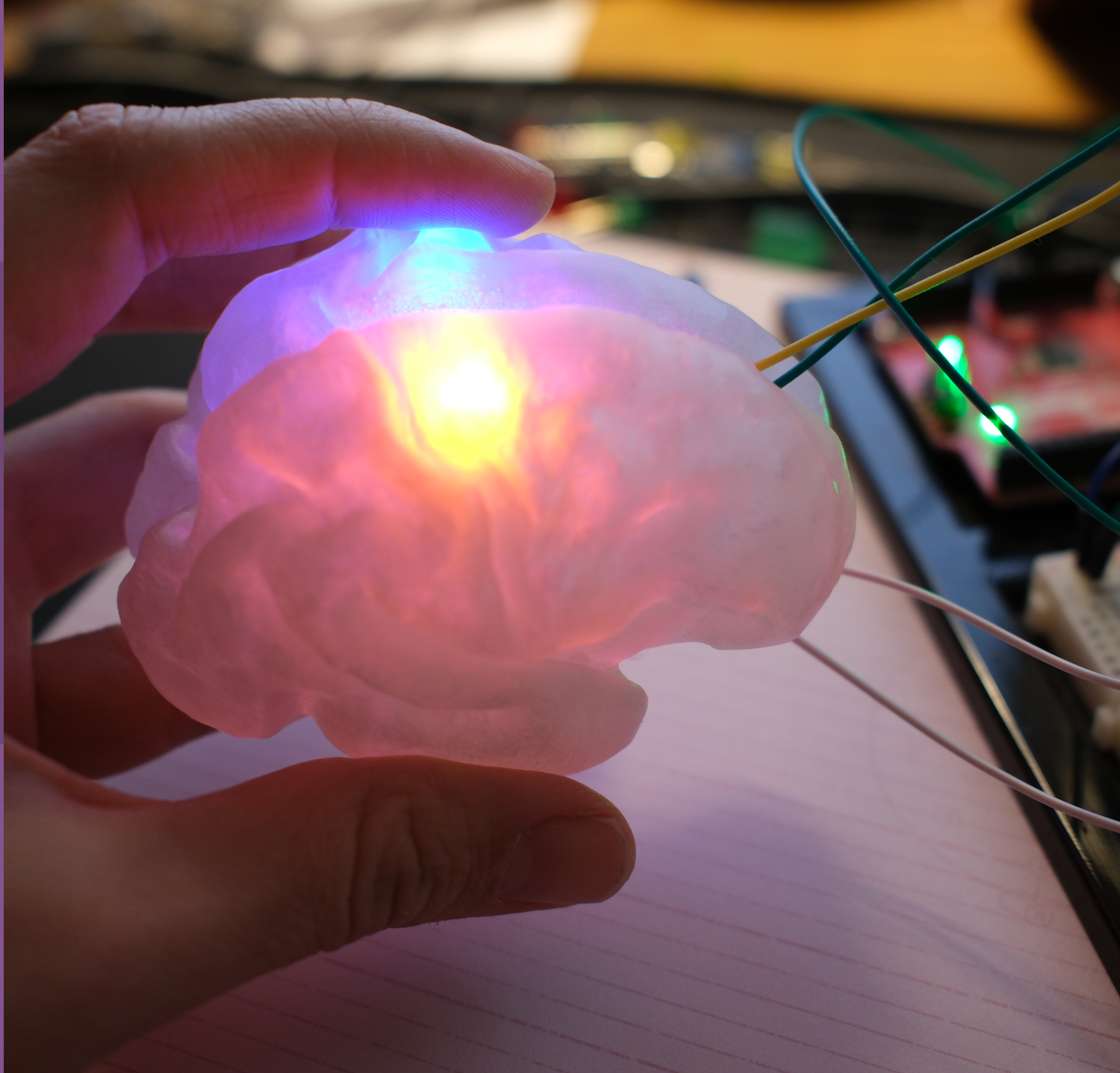
Spectral & Time-Frequency Analysis

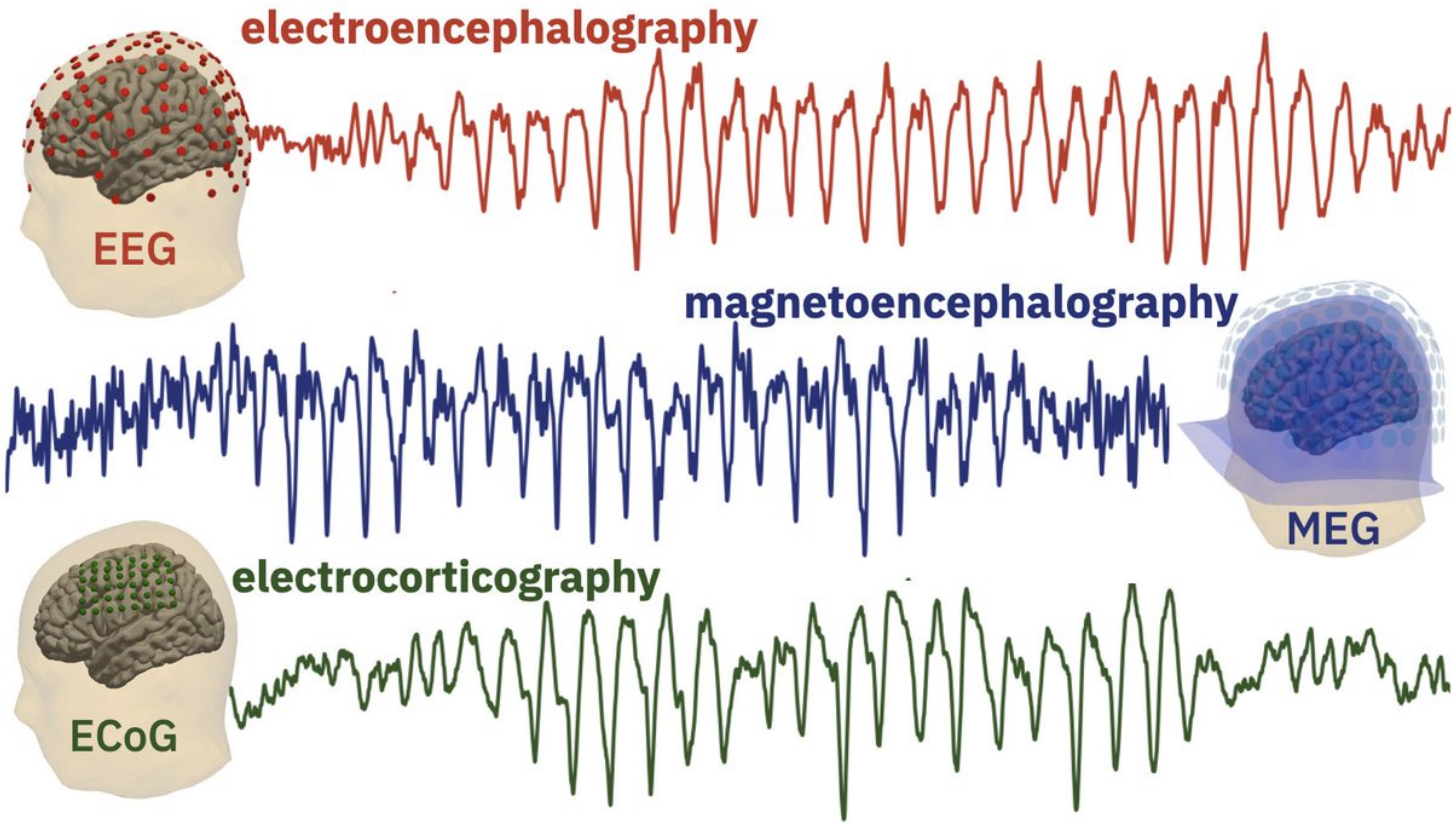


Natalie Schaworonkow

Oct 28th 2025

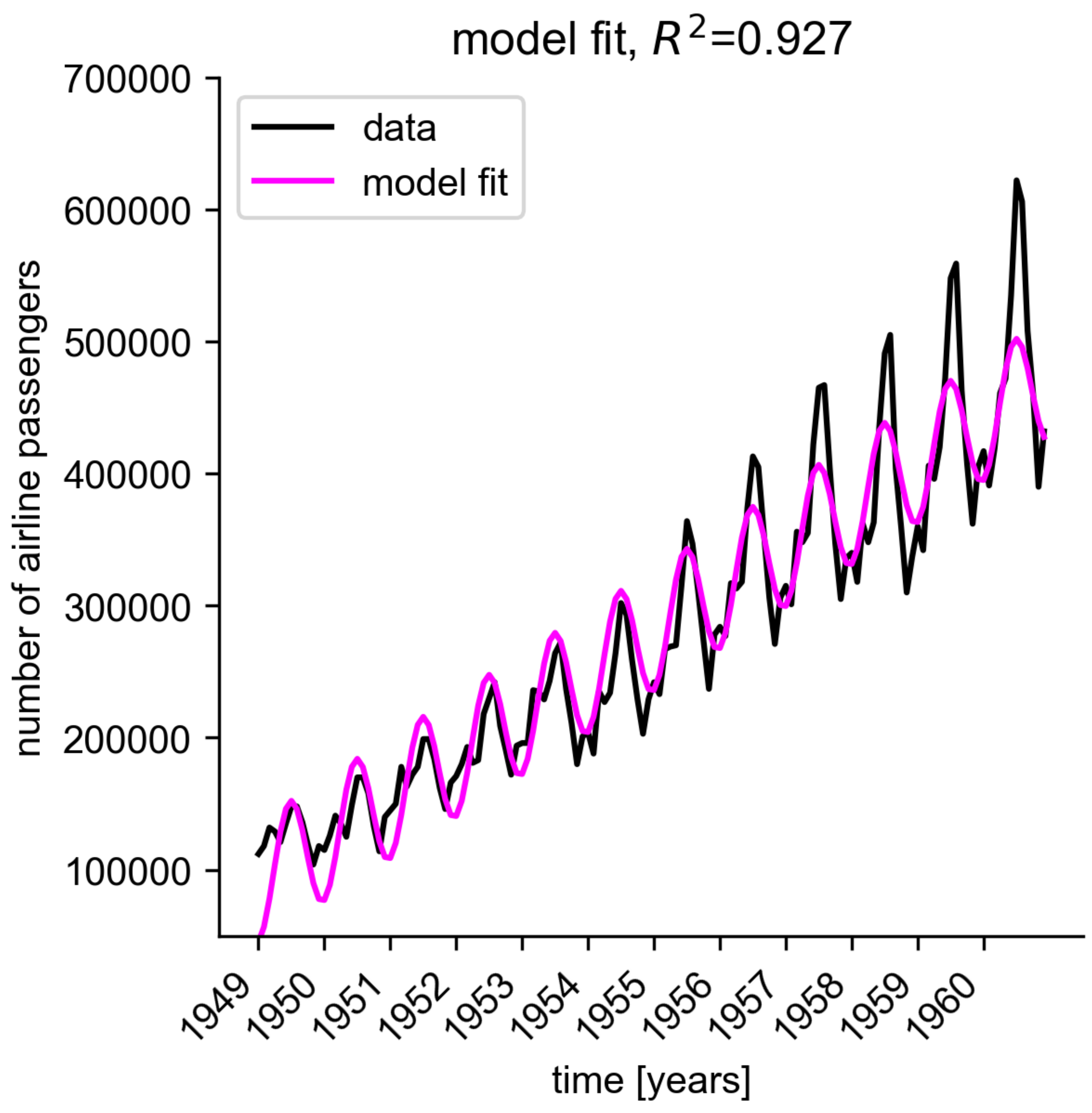
Outline

1. spectral analysis – computation
2. spectral analysis – interpretation
3. time-frequency analysis
4. volume conduction



spectral analysis – computation

motivation



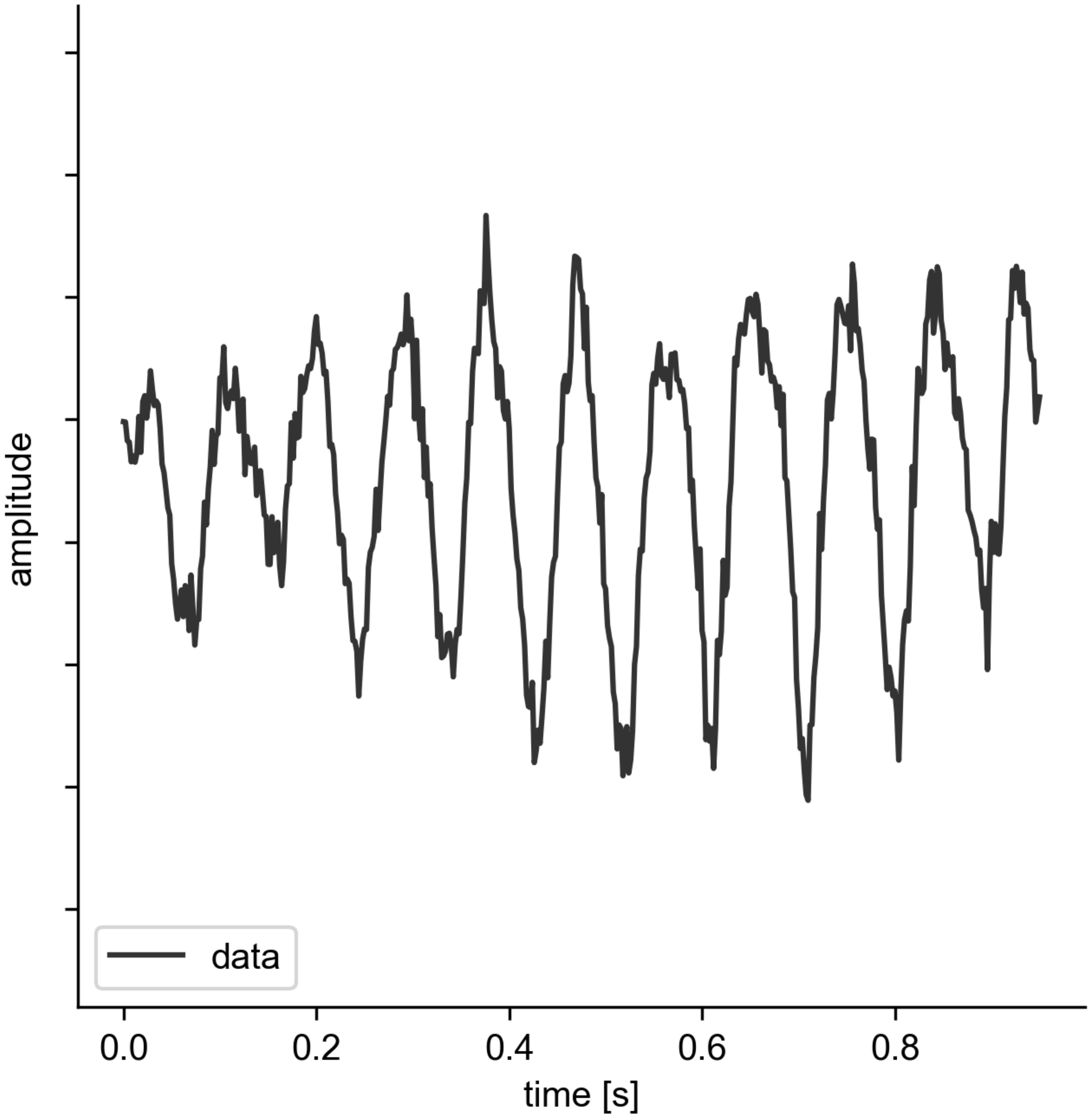
- how to detect systematic patterns in data?
- express measured data as a linear combination of several factors
- fit a model consisting of two factors
  - data ~ $coeff_1 * X_1 + coeff_2 * X_2$

    here: linear sine

purpose

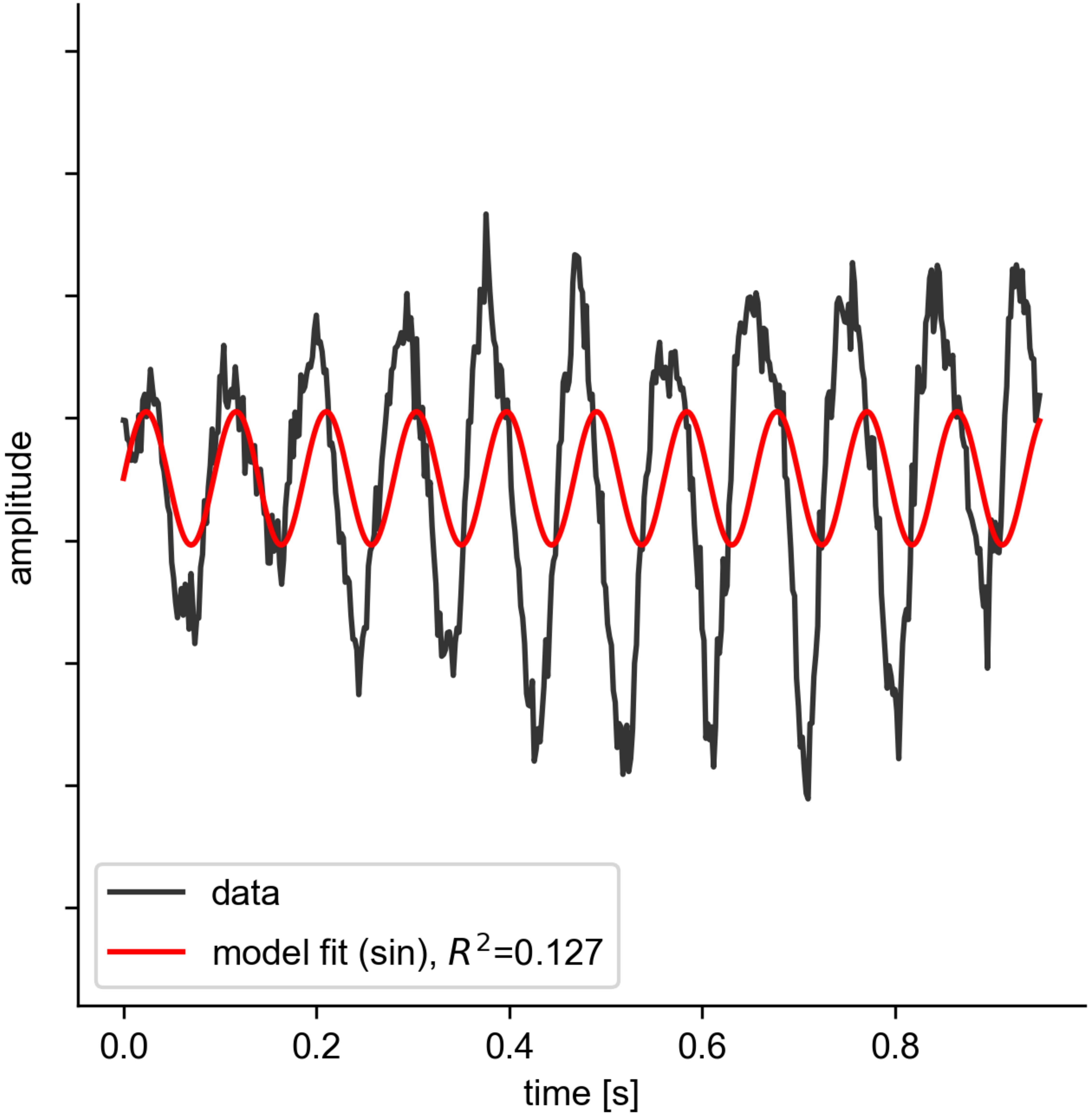
- description of data in an efficient way
- explain the data

data + model fitting



- 1 second of EEG data (500 data points)

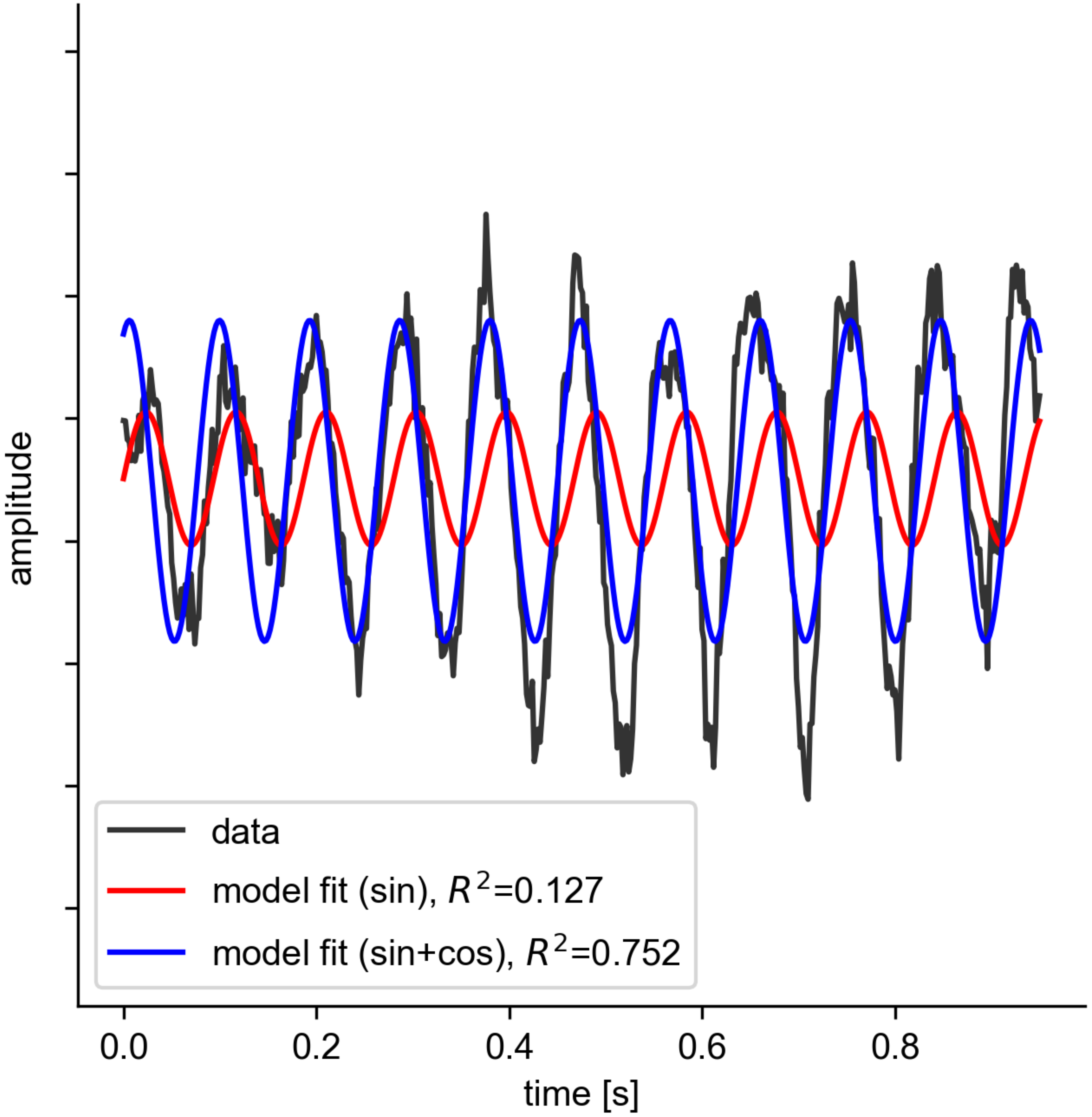
data + model fitting



- 1 second of EEG data (500 data points)
- **attempt 1**:
  model fit with 1 sine with fixed frequency

  data ~ $coeff_1$ * $sine_1$

data + model fitting

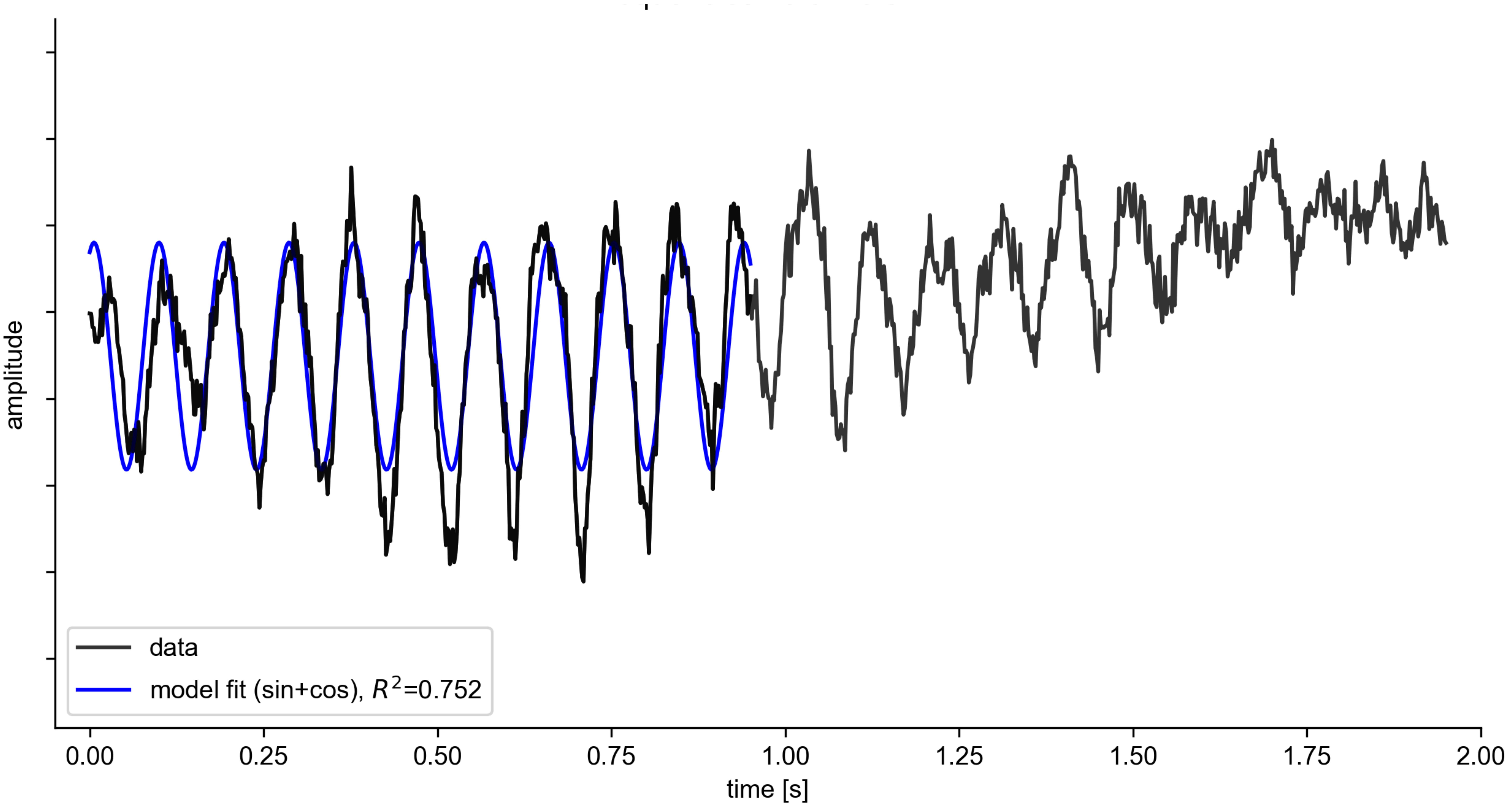


- 1 second of EEG data (500 data points)
- **attempt 1**:
  model fit with 1 sine with fixed frequency

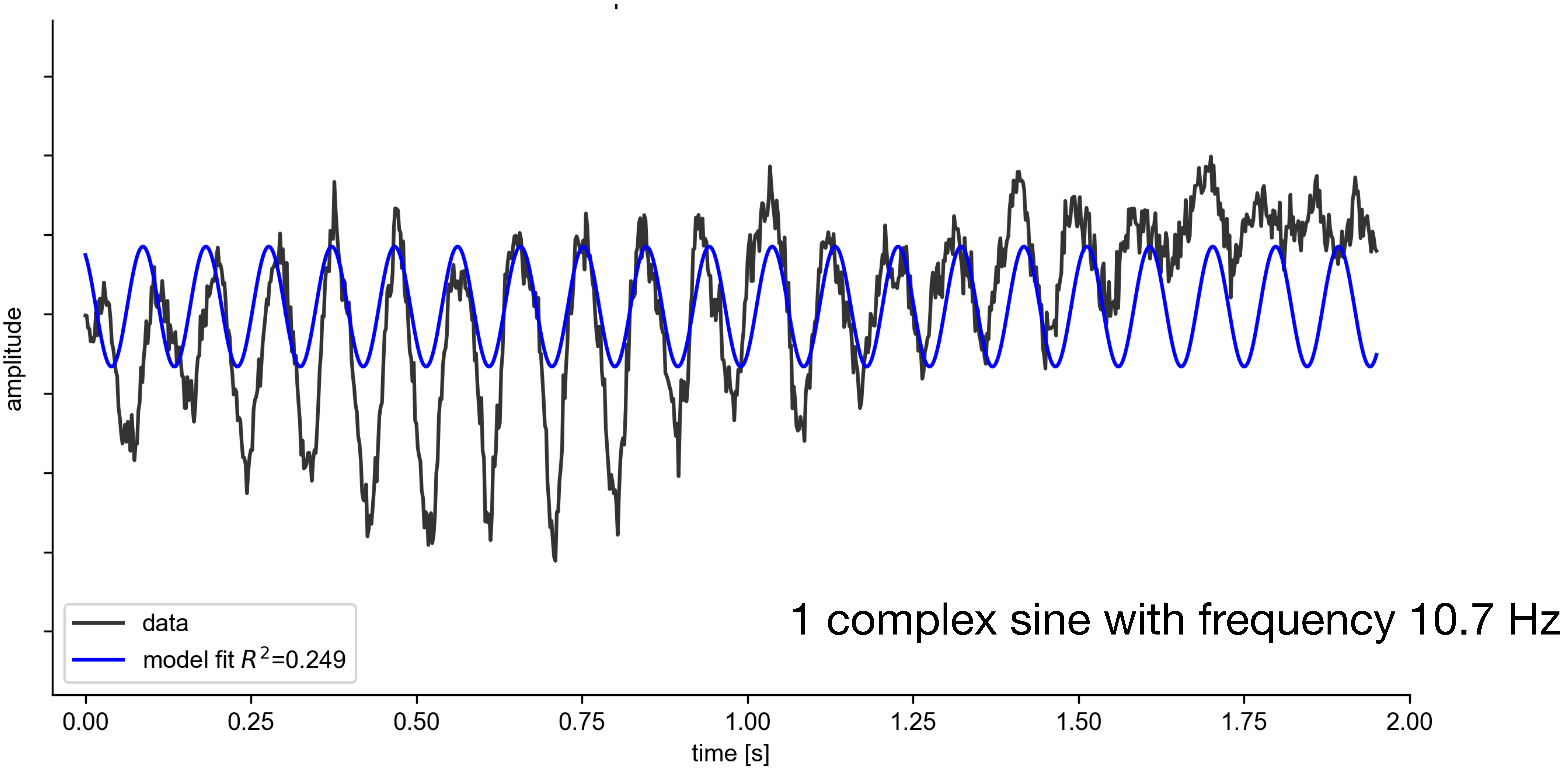
  data ~ coeff_1 * sine_1
- **attempt 2**:
  model fit with 1 complex sine (sine + cosine) with fixed frequency

  data ~ coeff_{1A} * sine_1 + coeff_{1B} * cosine_1

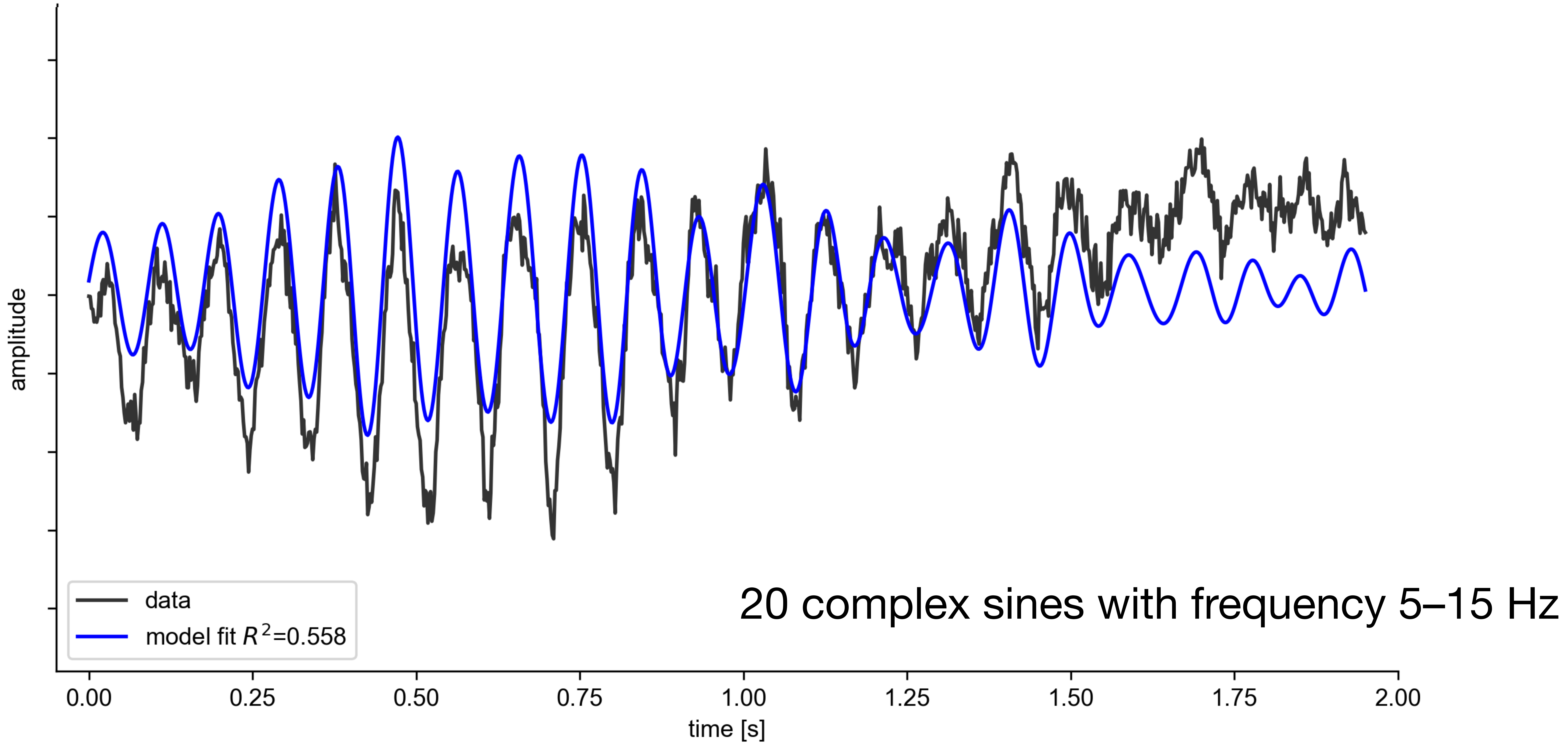
data + model fitting



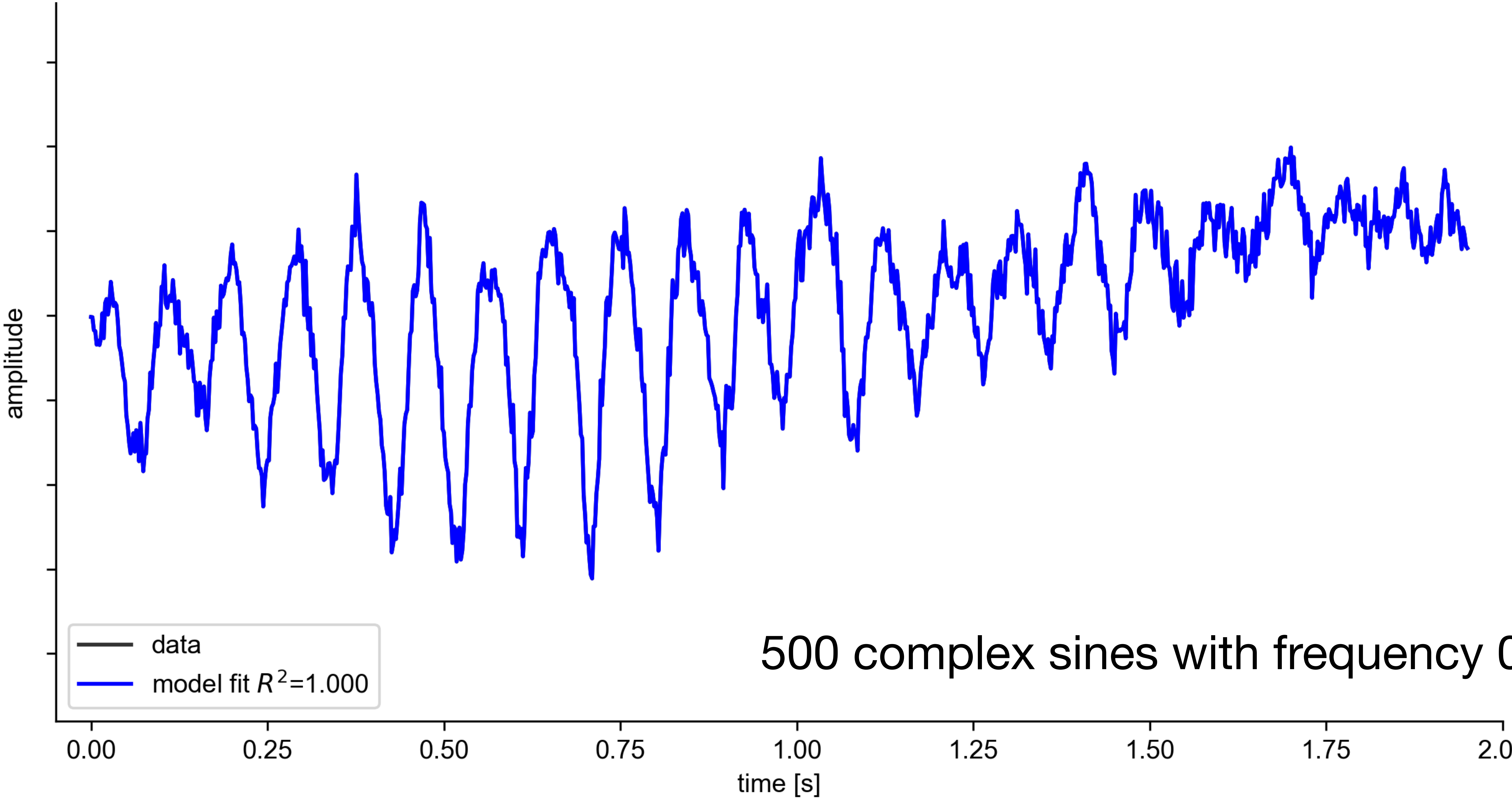
data + model fitting – more data



data + model fitting – more data



data + model fitting



500 complex sines with frequency 0–250 Hz

concepts / different views

input
time series, N samples

computation →

output
N coefficients for N/2 sinusoids

two views on the same procedure

statistical modeling

- “multivariate linear model with complex sinusoids as regressors”
- no degrees of freedom, no remaining residual → overfitting

signal processing

- “Fourier transform”, a transform into the spectral domain
- signal representation, perfect reconstruction possible

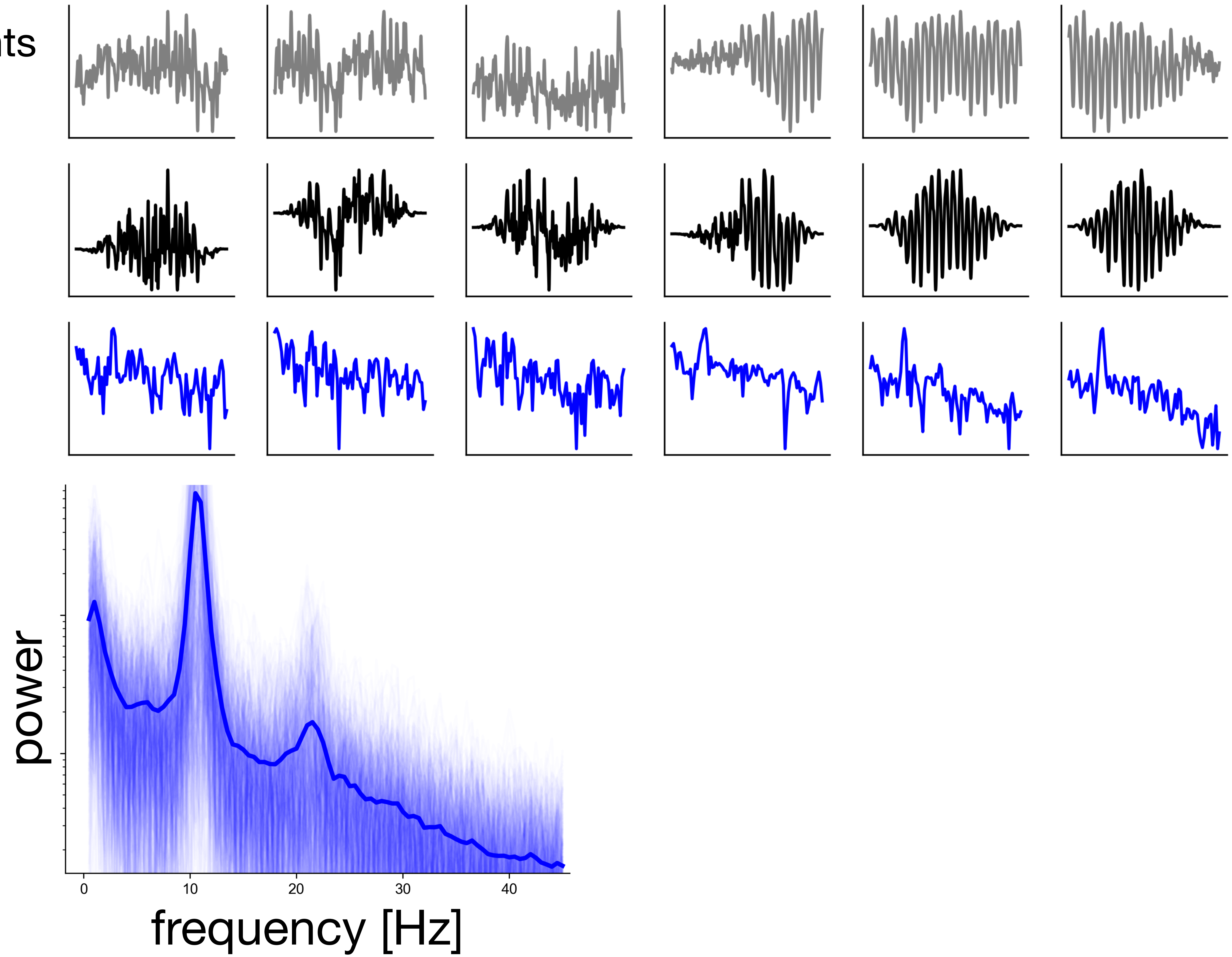
computation: Welch’s method

time series, cut into small segments
[nr_segments x nr_samples]

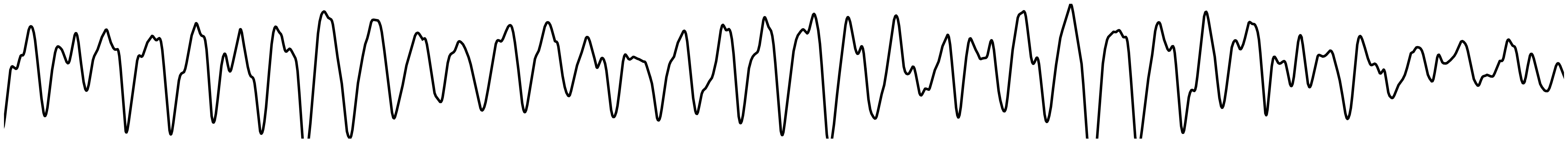
time series, multiplied with a windowing function
[nr_segments x nr_samples]

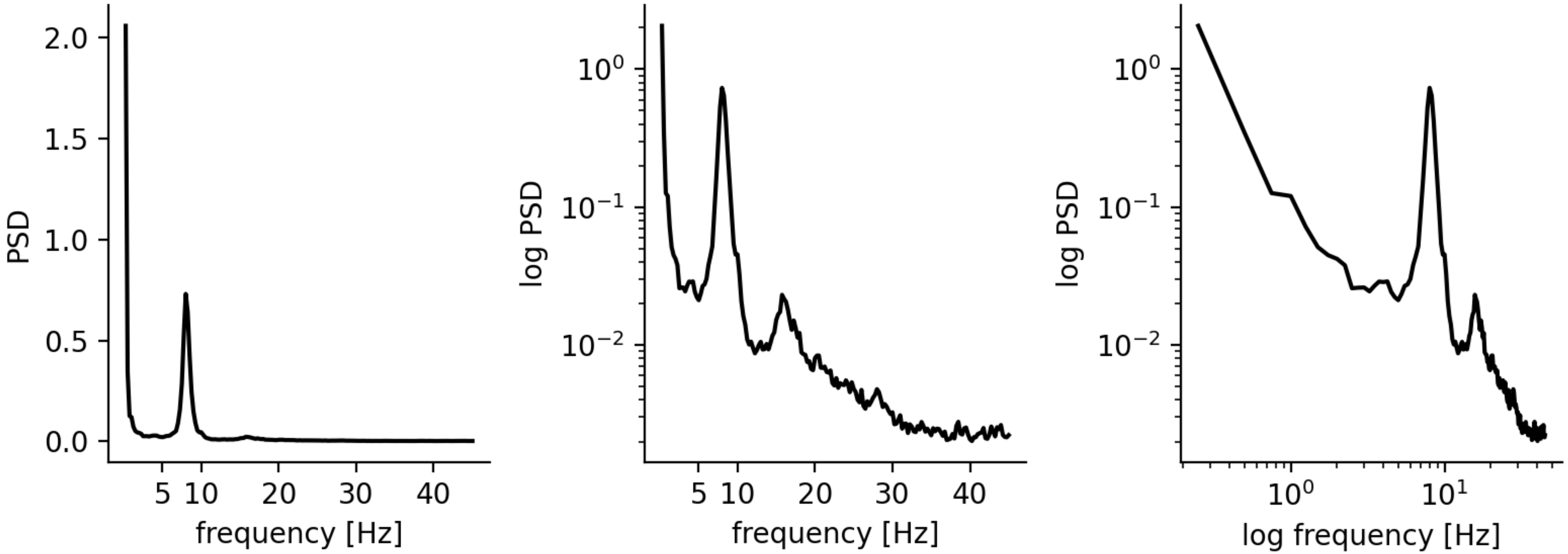
spectra for each segment
[nr_segments x nr_frequencies]

average across frequencies to obtain the spectrum
[1 x nr_frequencies]



example output for real EEG data





linear-linear | linear-log | log-log

diffferent ways to show the same output of the spectral estimation

methods: parameters in general

common spectral analysis parameters

- length of data segments
- number of points per segment
- percentage of overlap
- type of taper

What are the “correct” parameters?

Choice depends on analysis goal. Different parameters enable flexibility in analysis.

if in doubt:

- starting point: take the ones from your elders / previous literature
- change parameters and observe effects, effects should be robust for similar parameters → sensitivity analysis
- simulate!

methods: frequency resolution

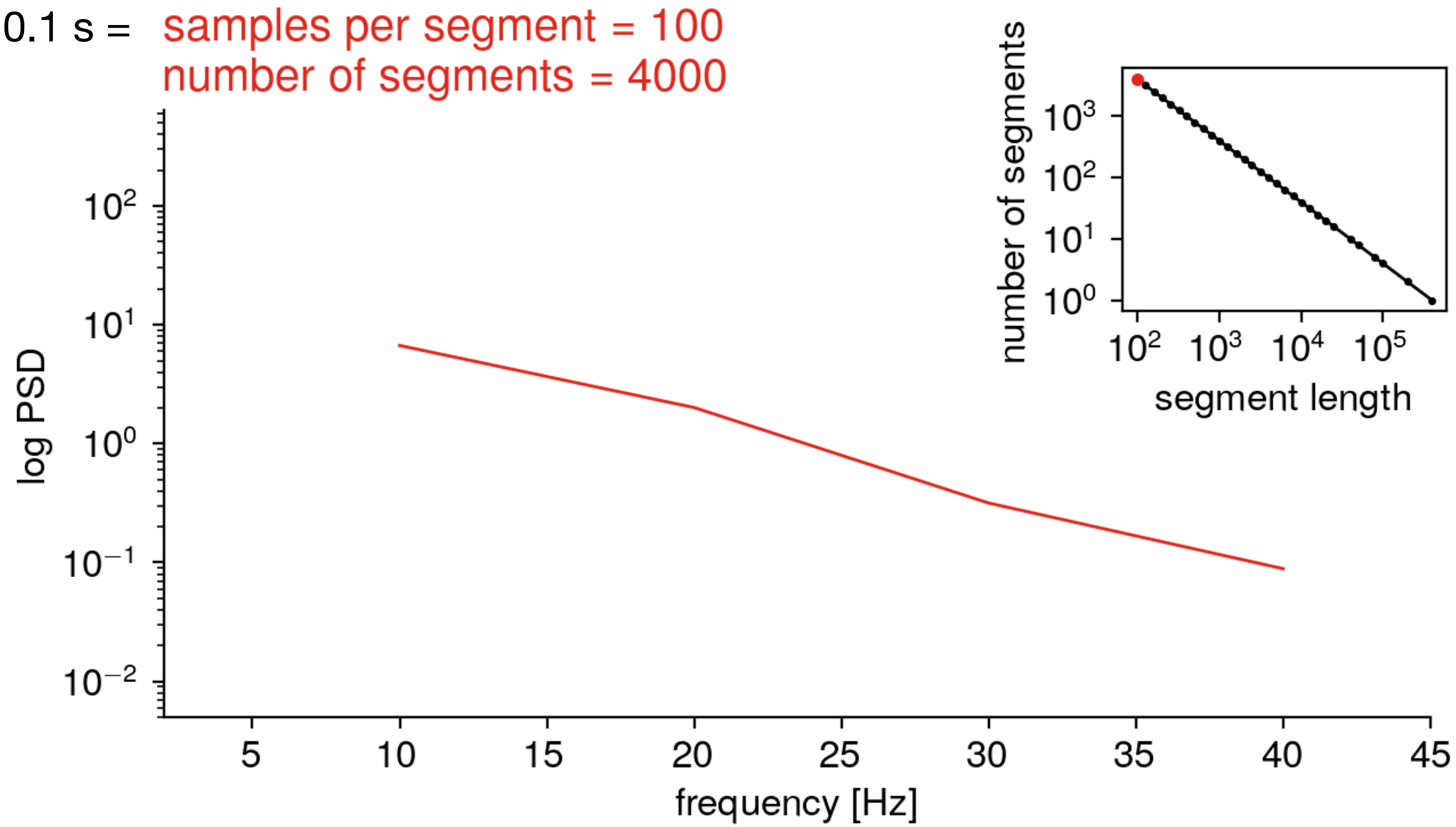
- frequency resolution of the spectrum depends on the number of used samples for each segment

Example

		frequency resolution					
• segment length:	`1s` →	`1`	`2`	`3`	`4`	…	`Hz`
• segment length:	`2s` →	`0.5`	`1`	`1.5`	`2`	…	`Hz`
• segment length:	`0.1s` →	`10`	`20`	`30`	`40`	…	`Hz`

methods: frequency resolution

- changing the settings how many samples per segment are used determines frequency resolution.



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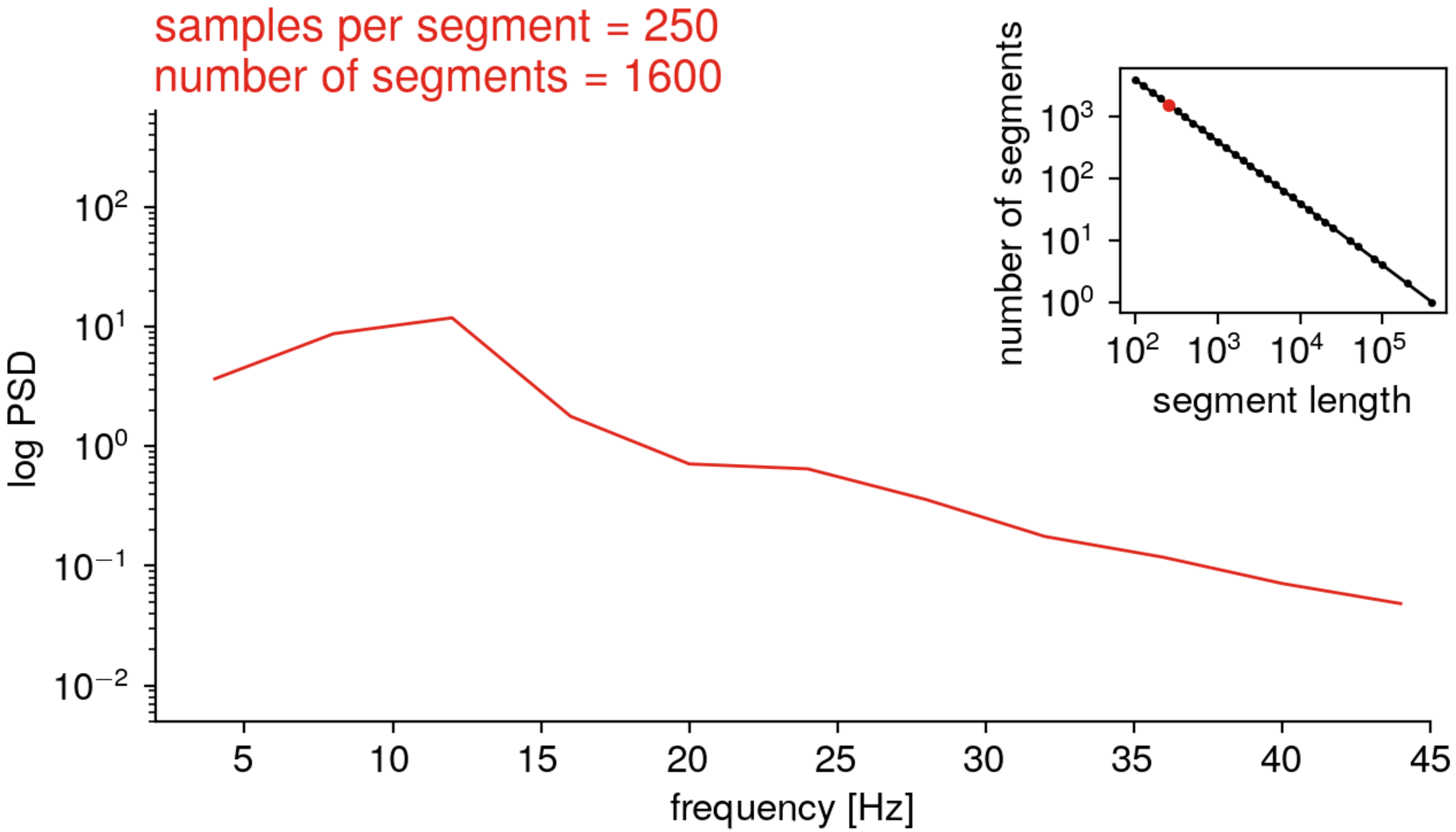

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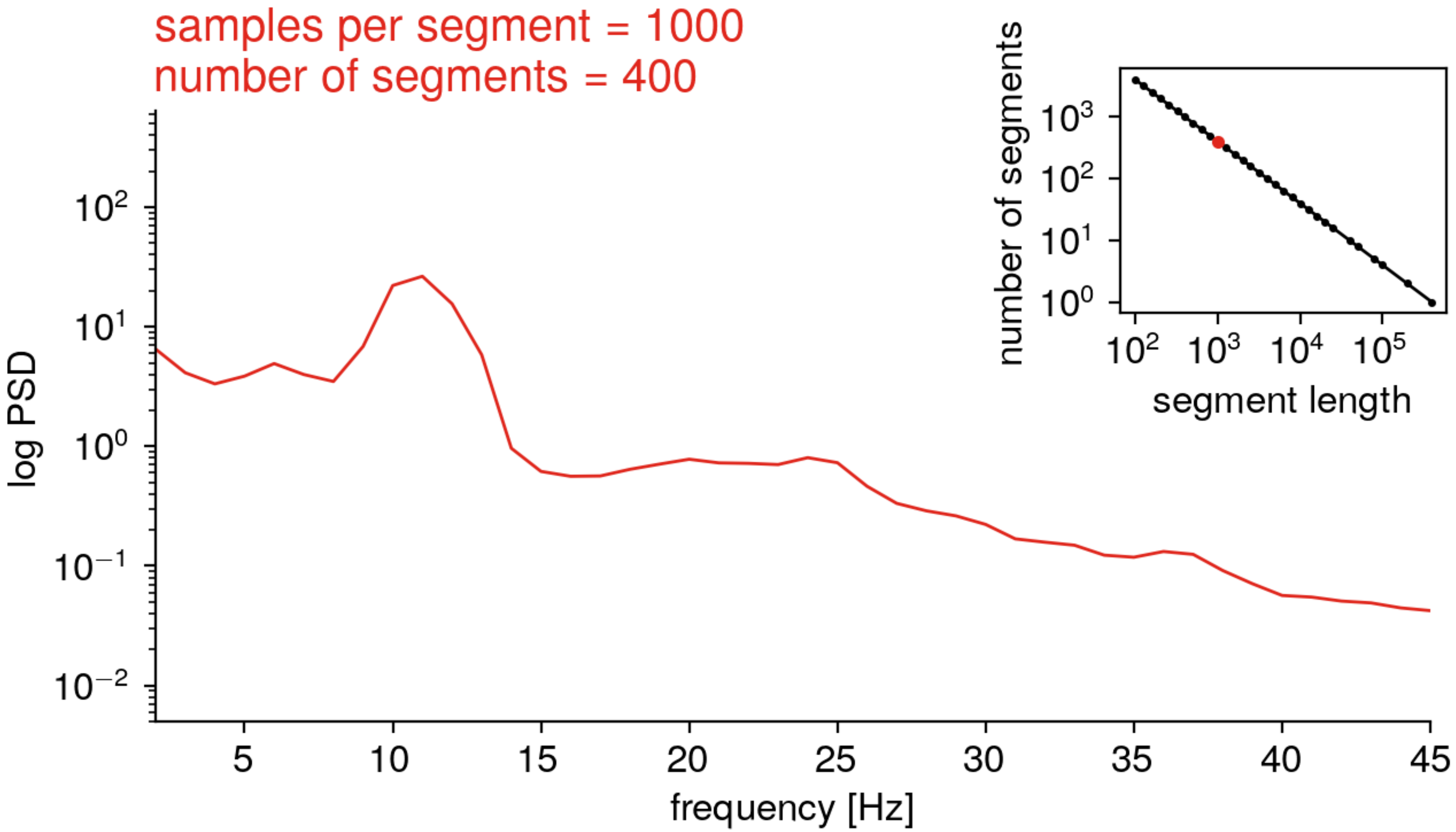

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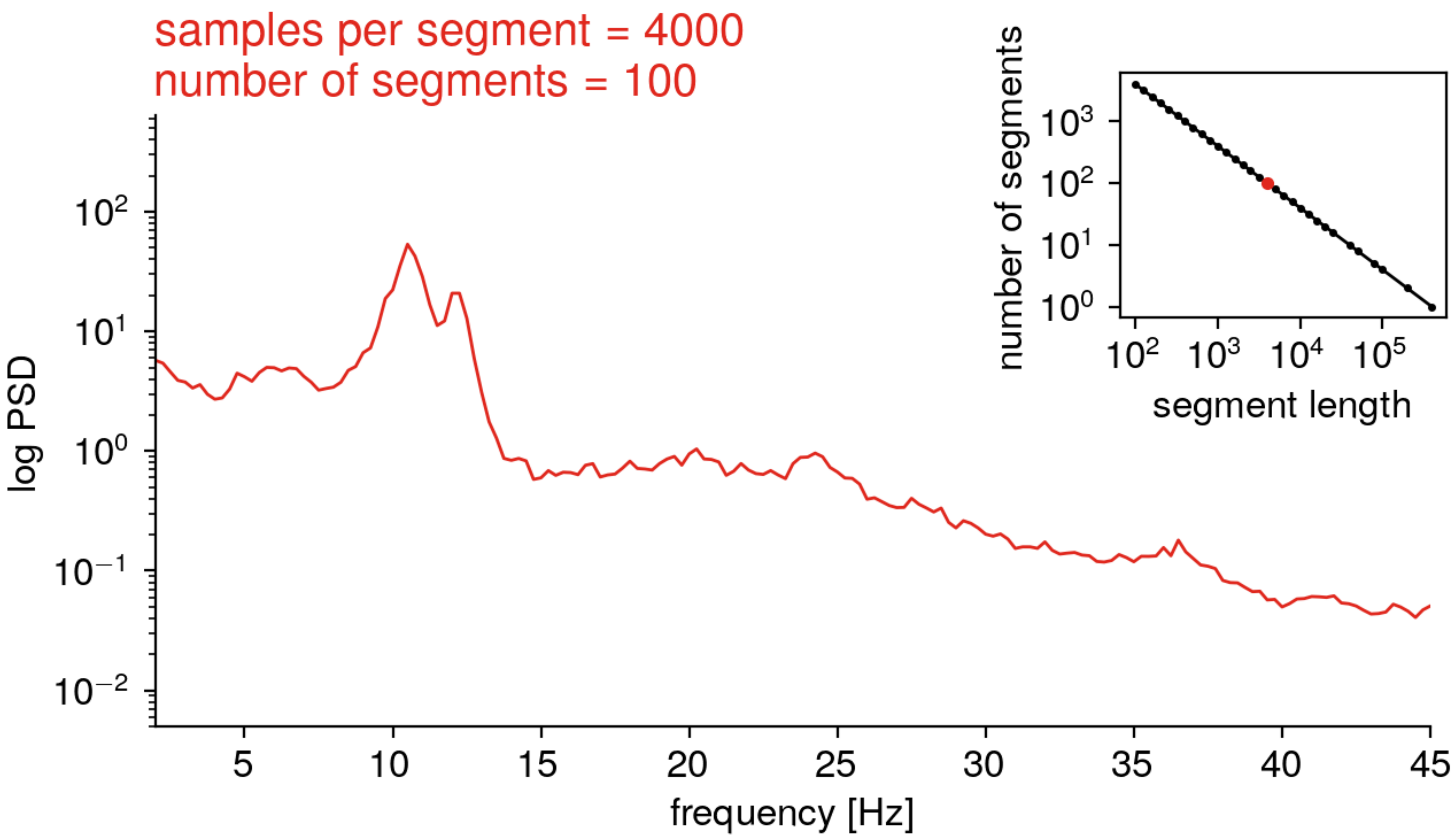

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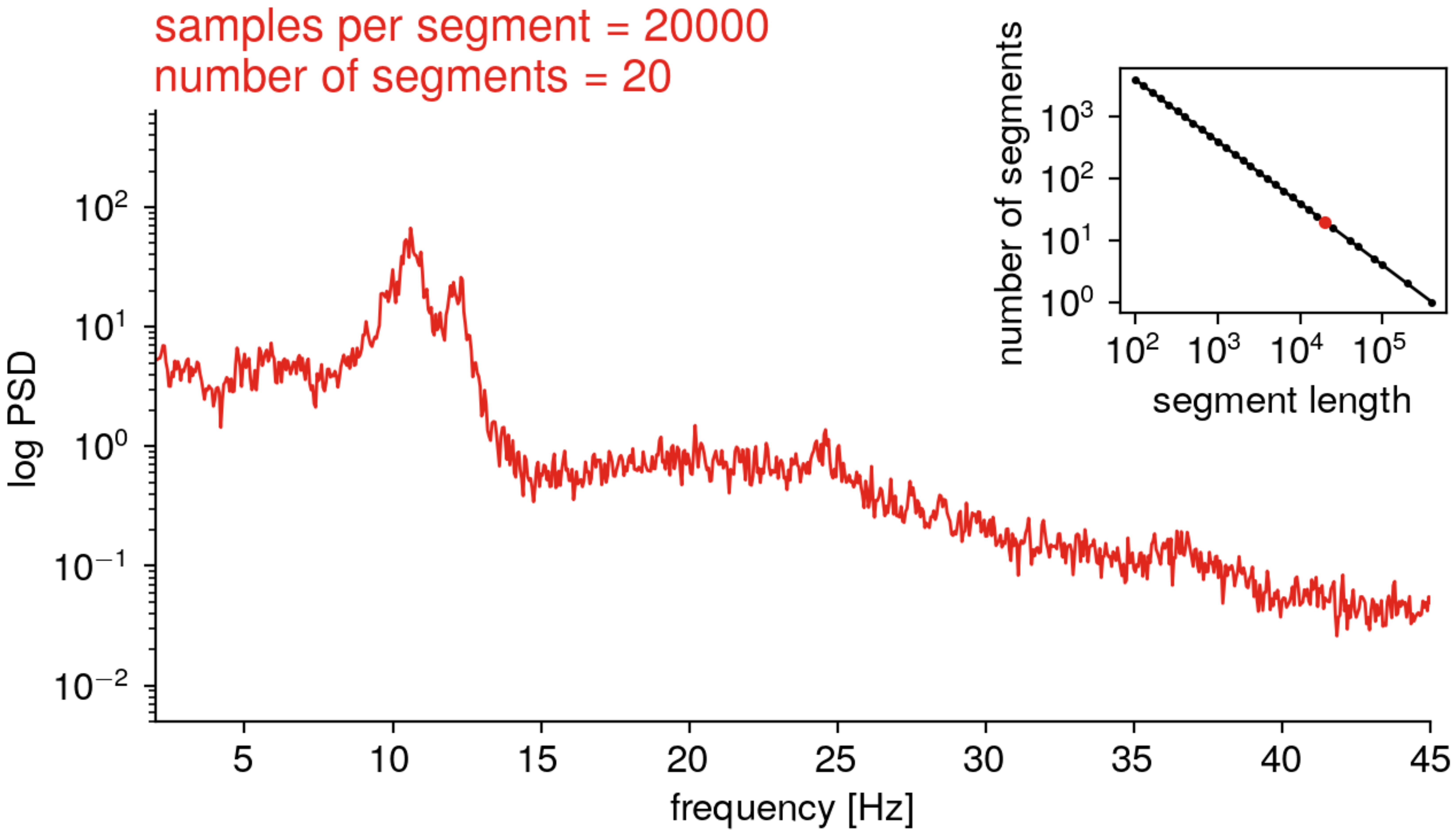

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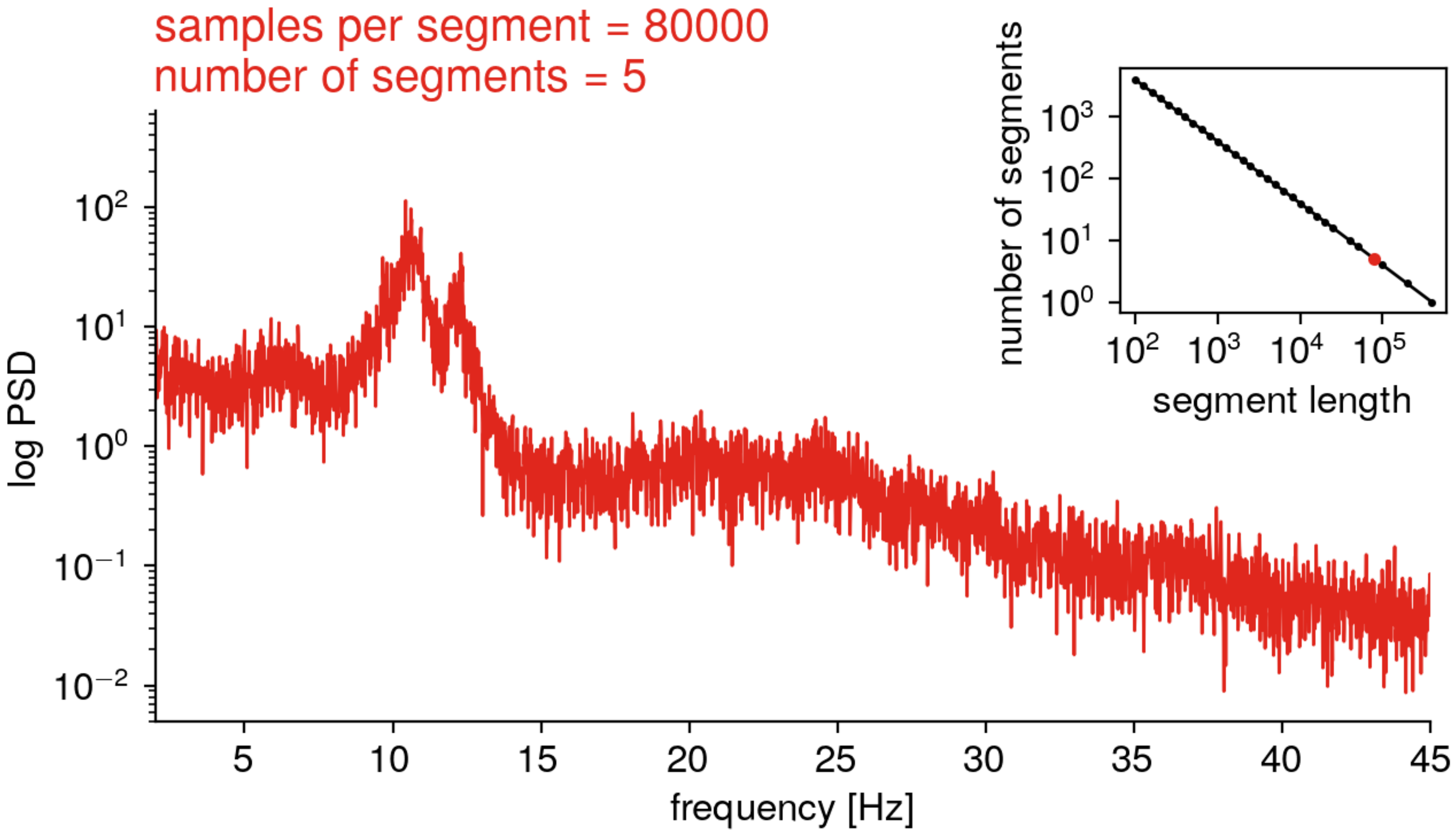

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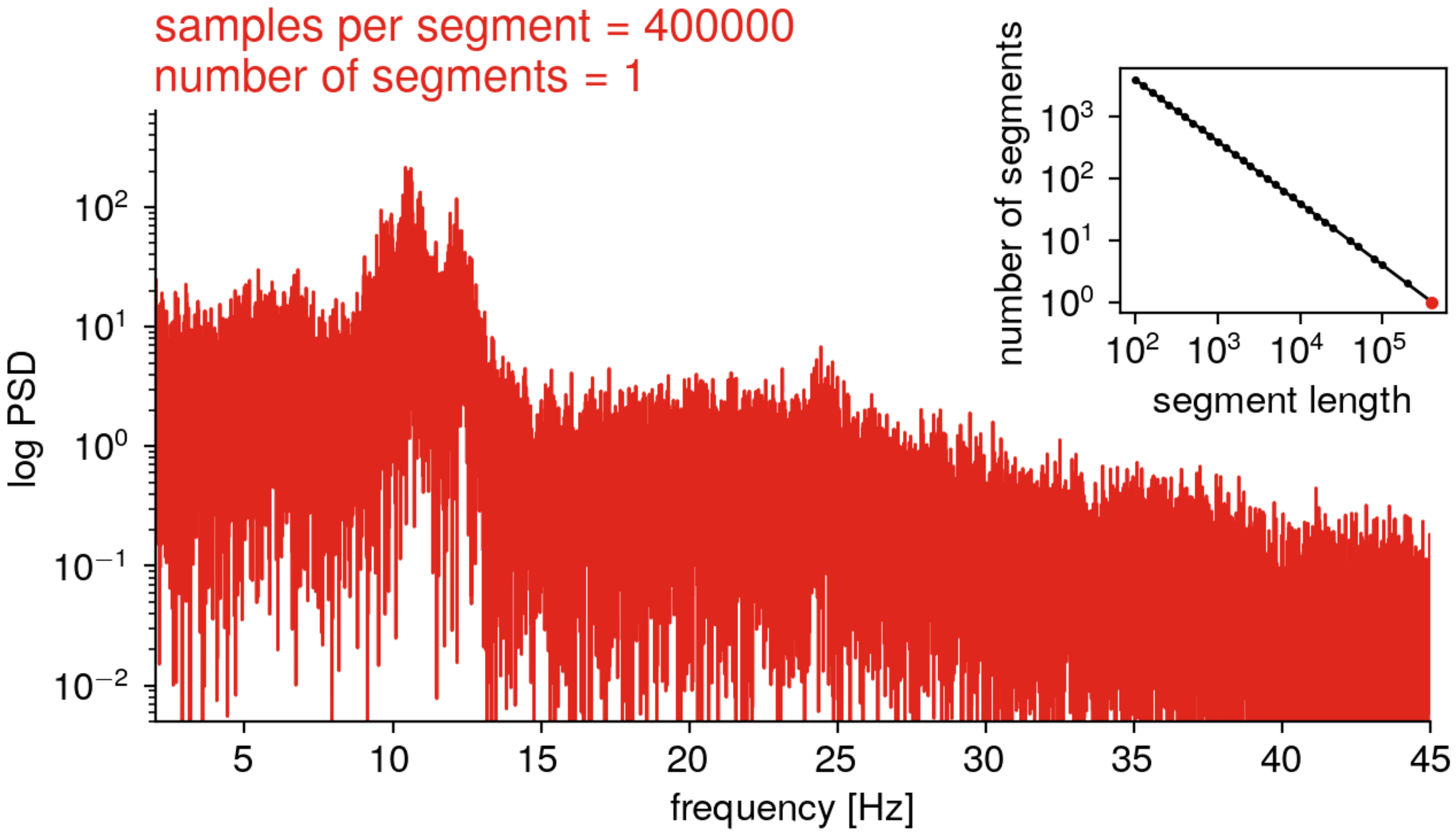

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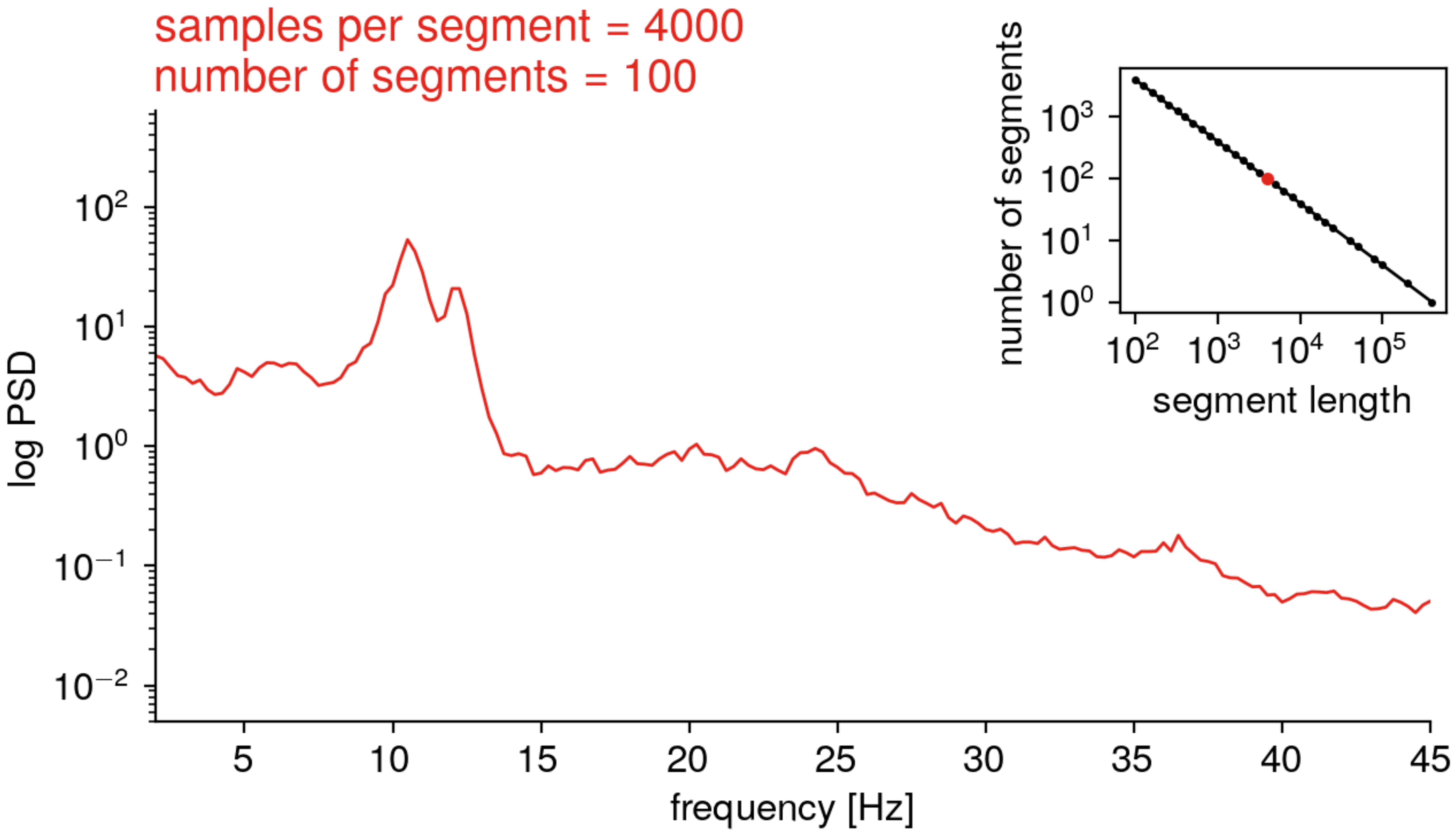
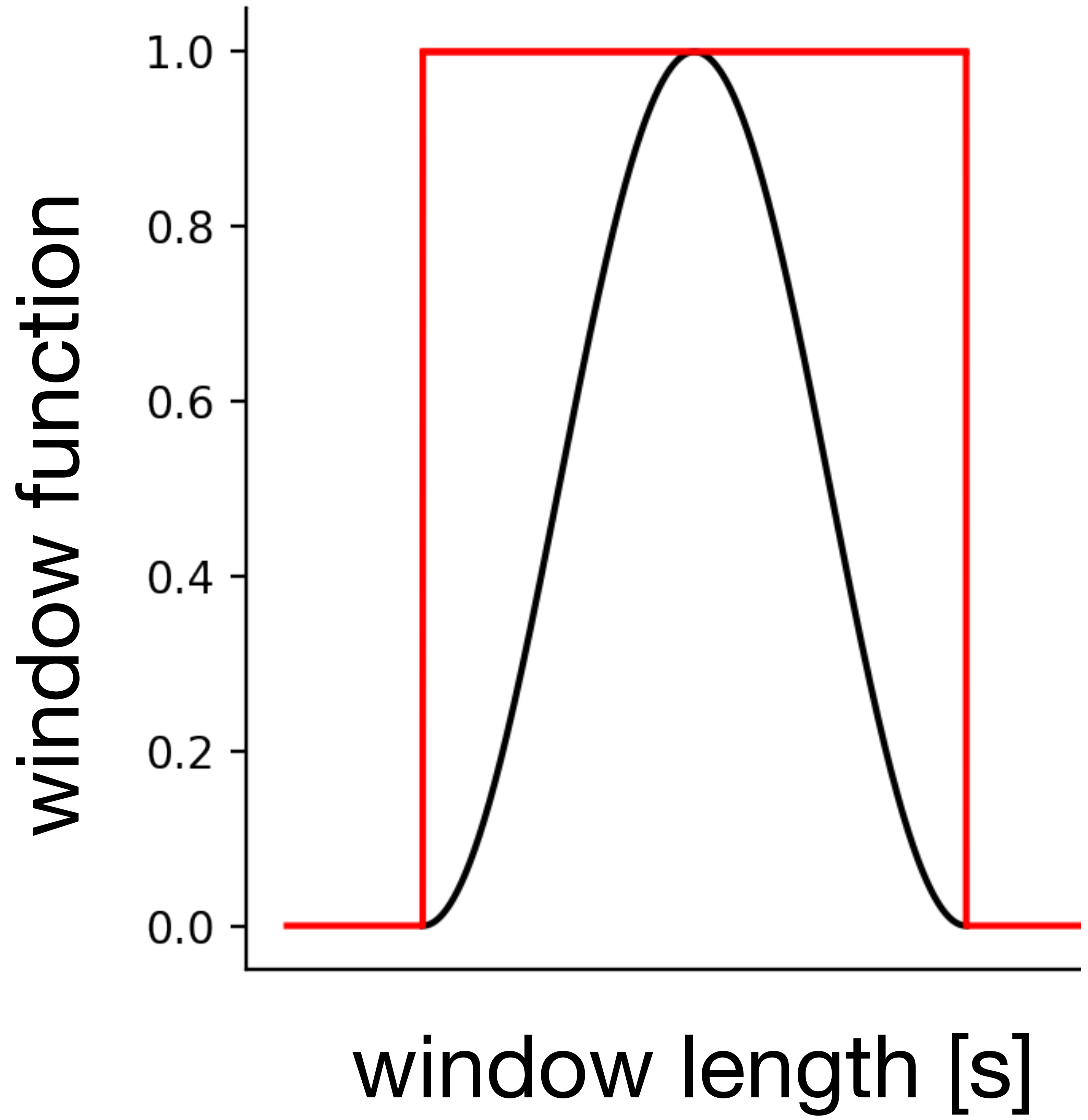
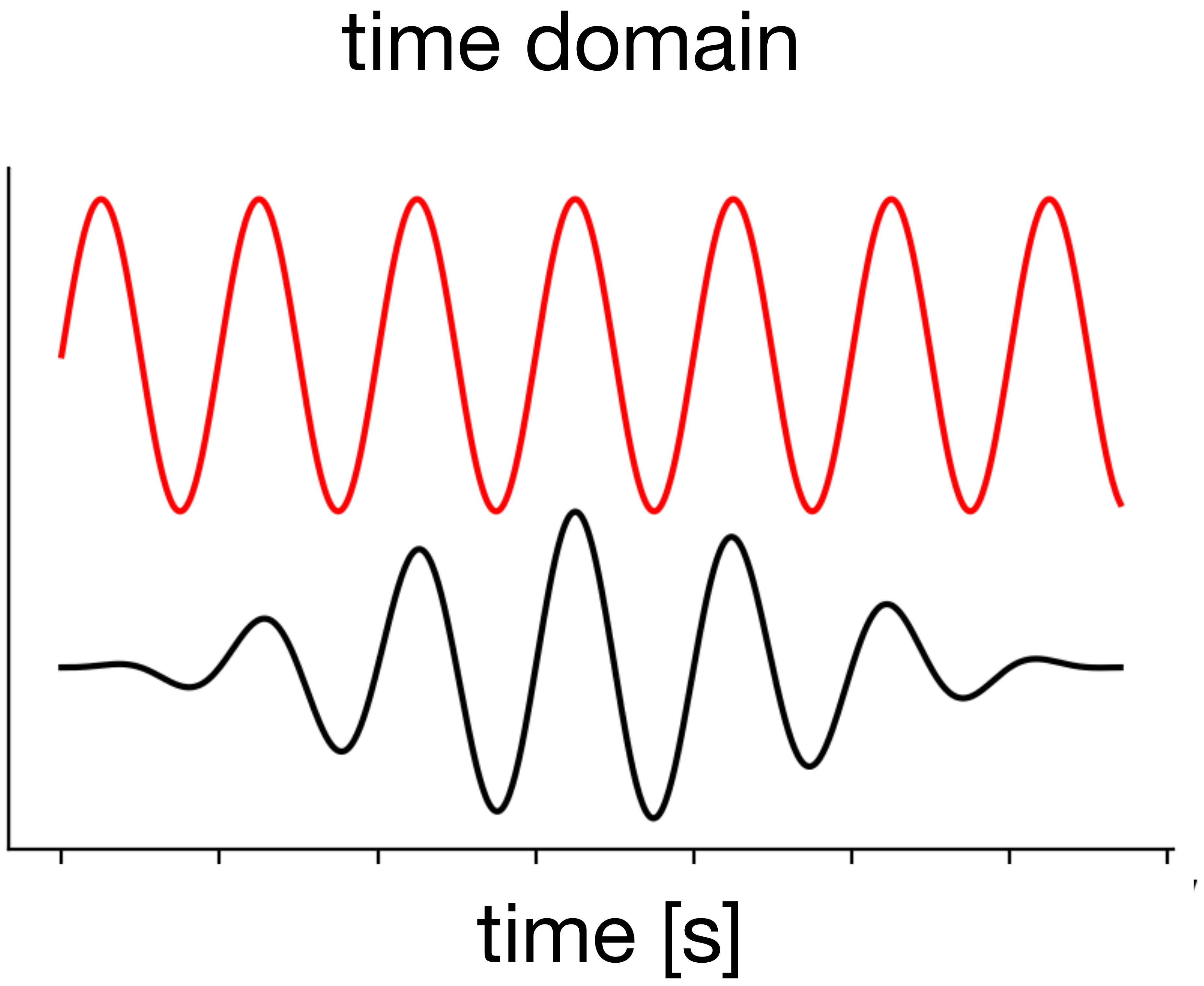


methods: taper / window function





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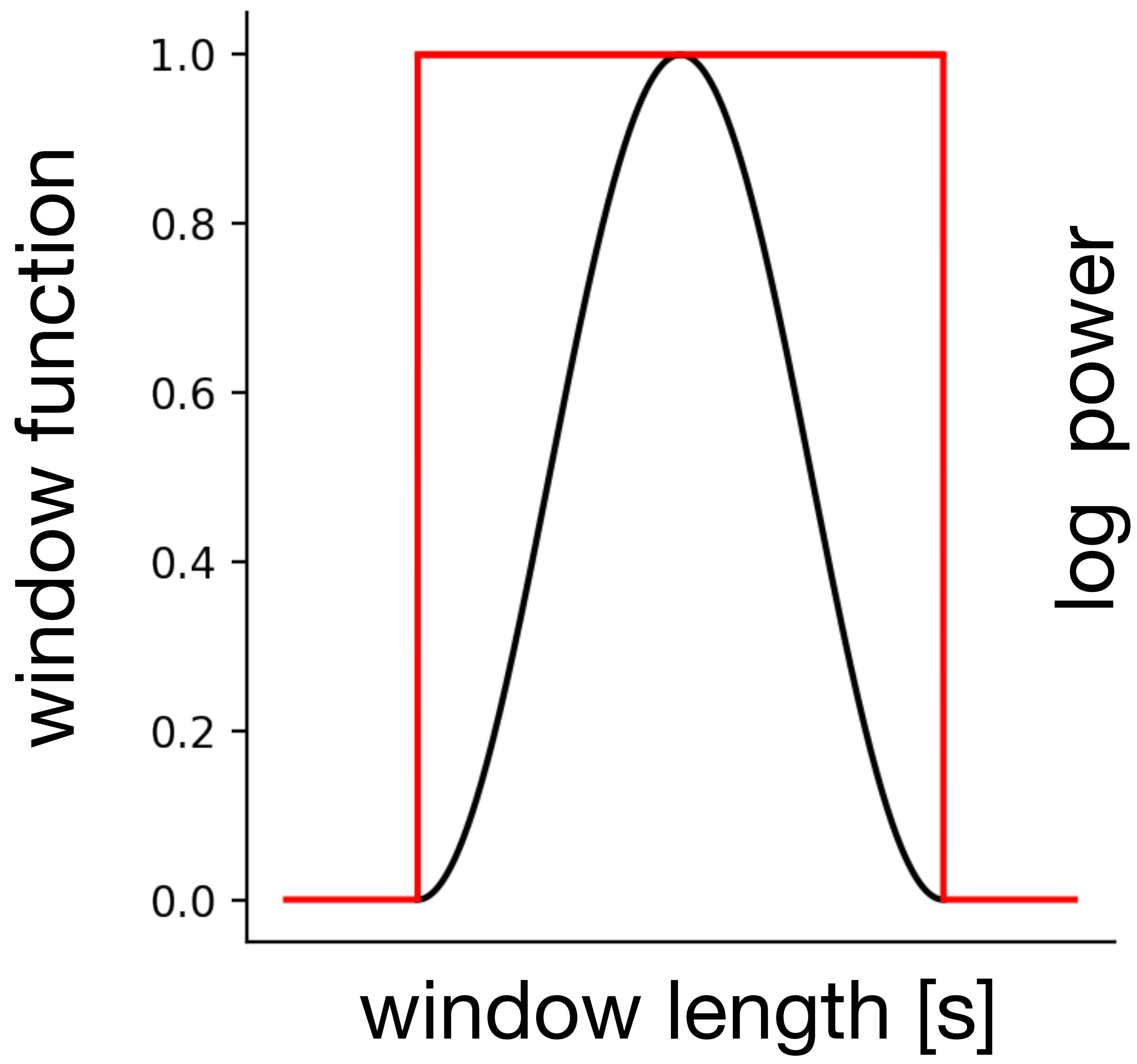
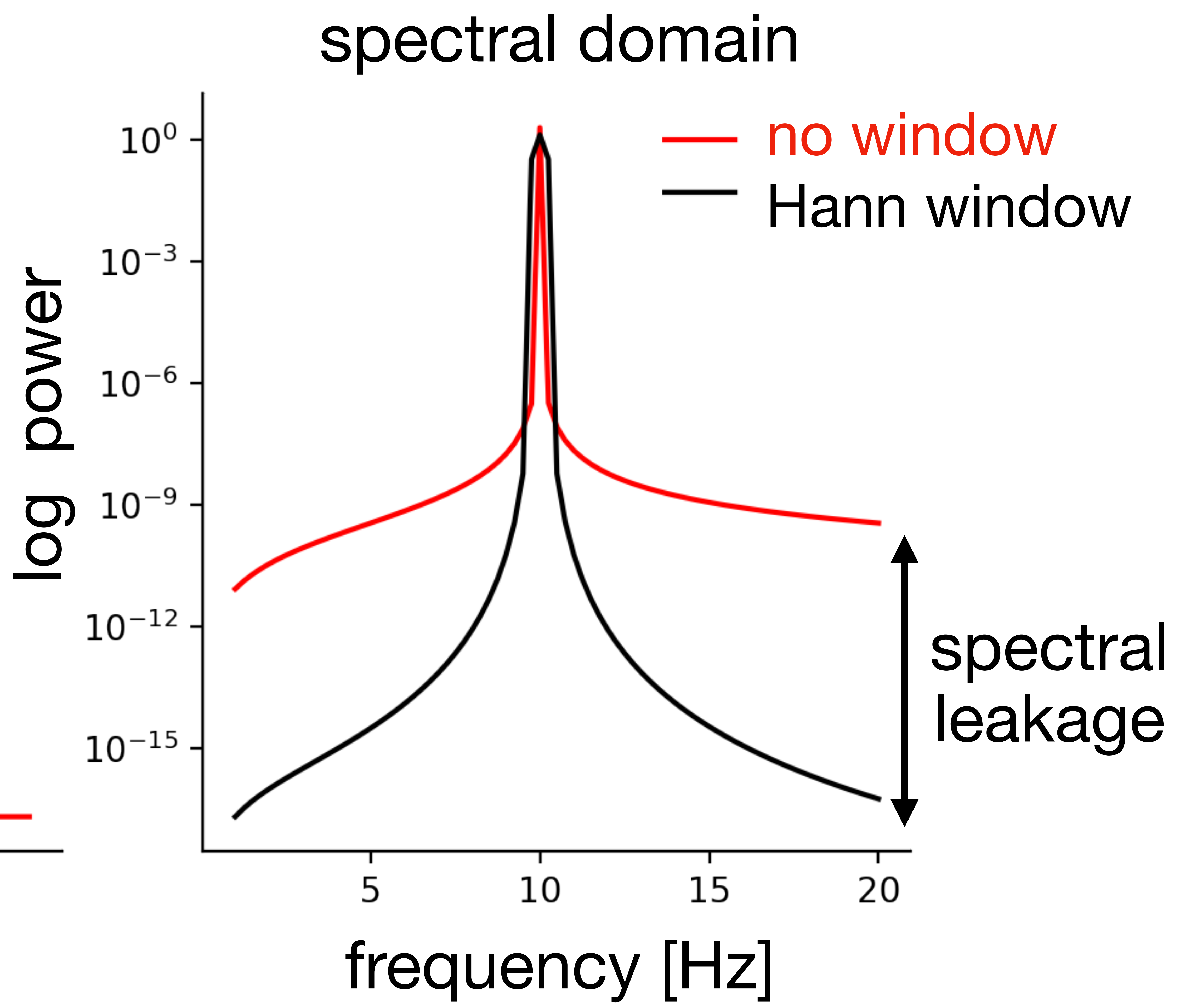




methods: multitaper, goal: increase SNR

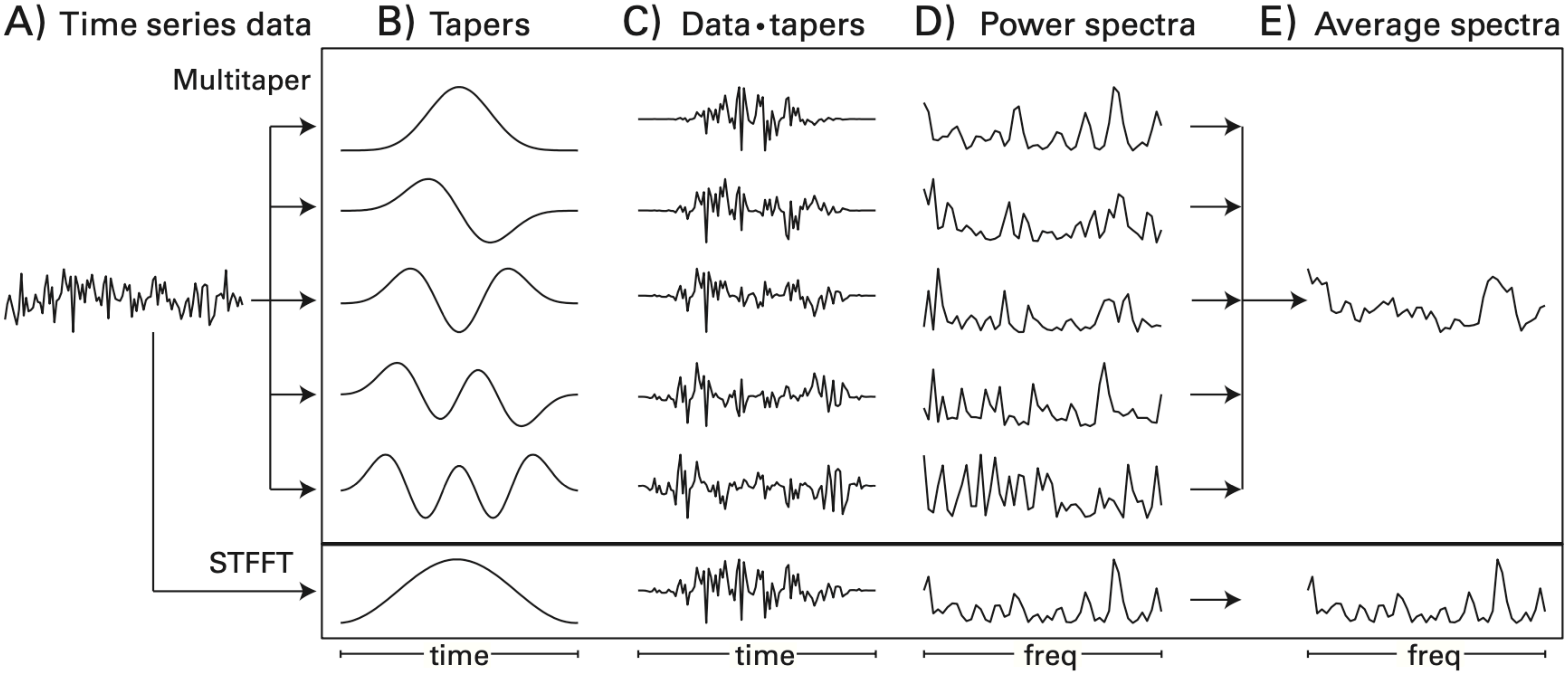
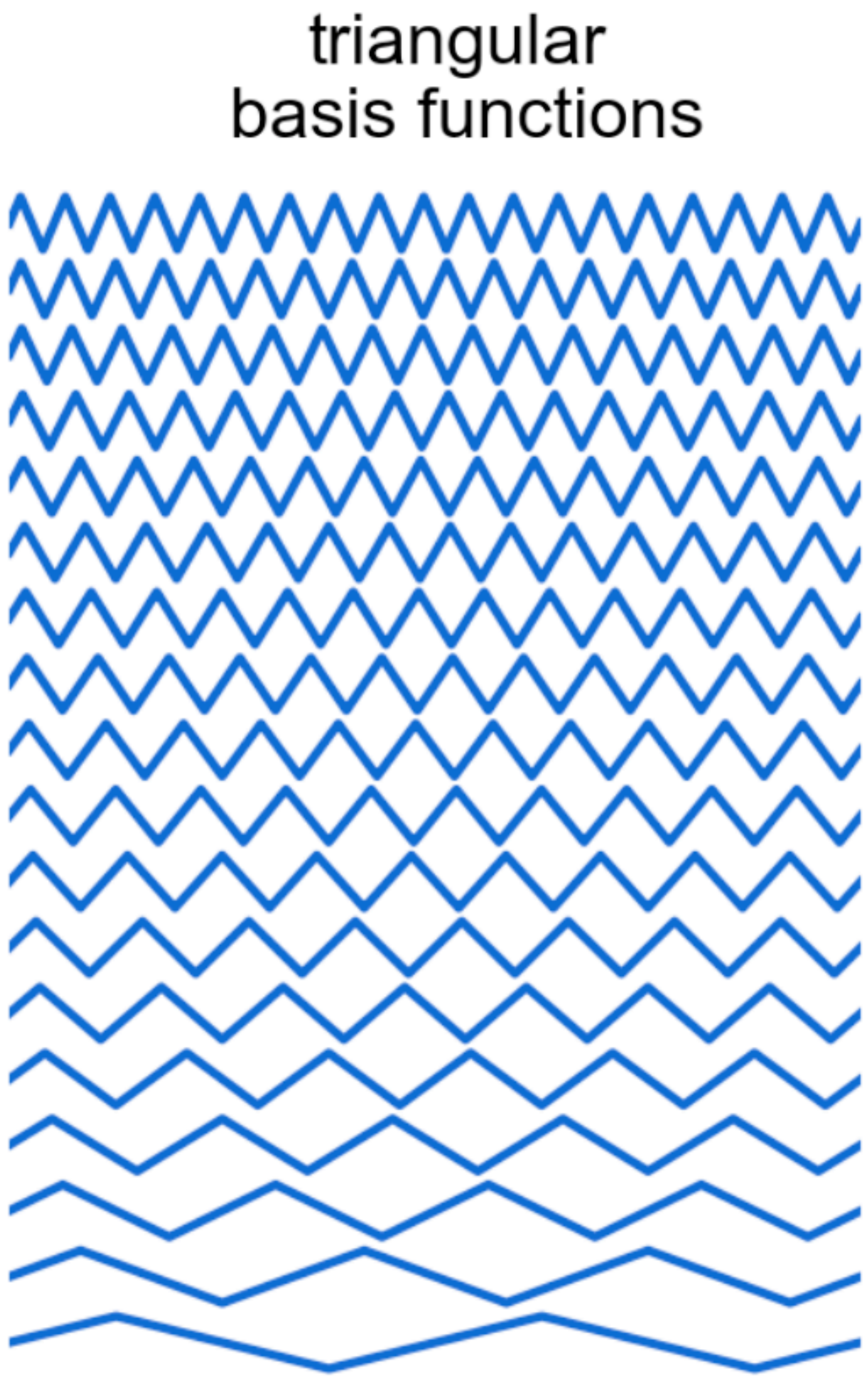
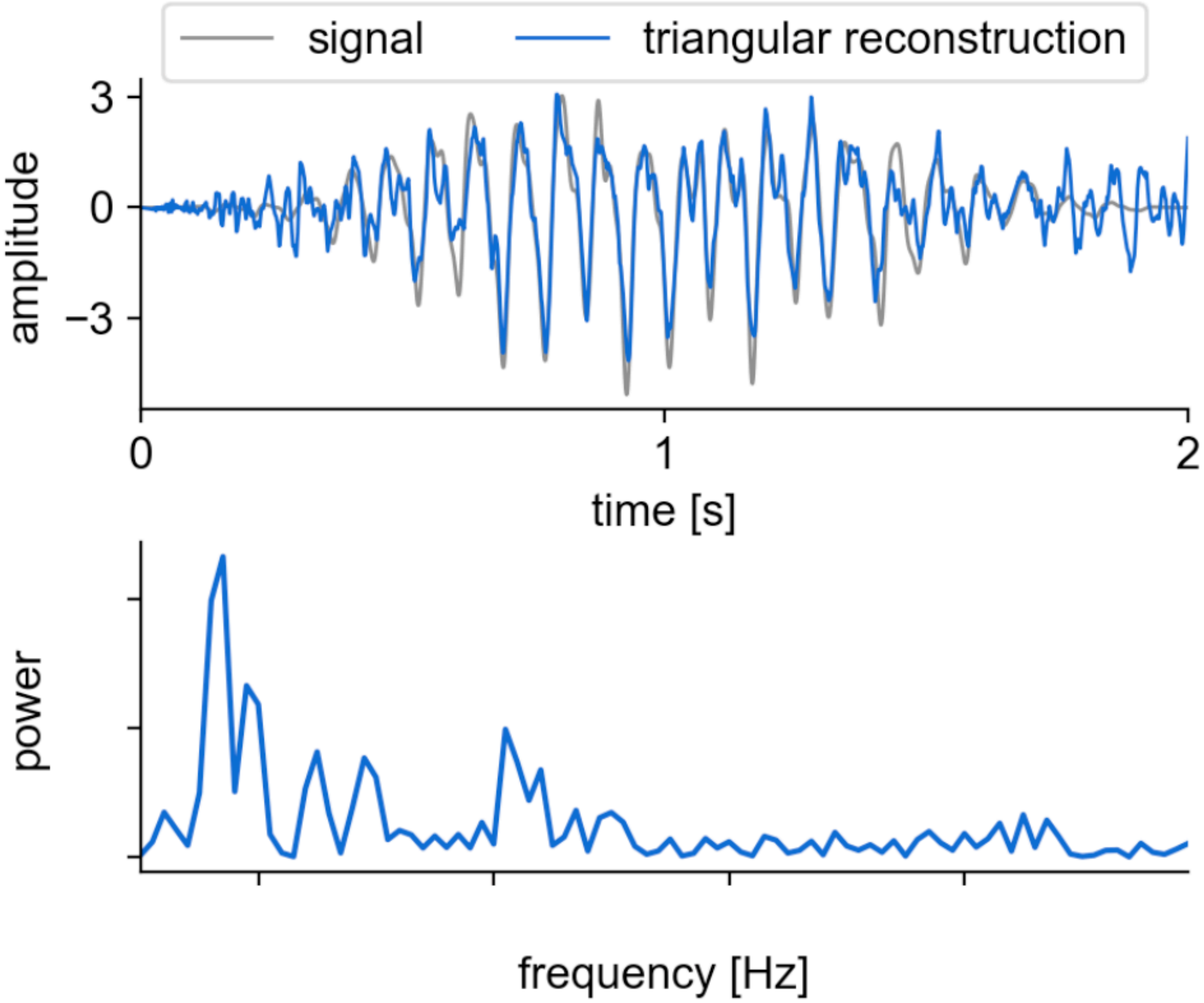


figure from: Mike X. Cohen – Analyzing Neural Time Series Data

other functions as factors?

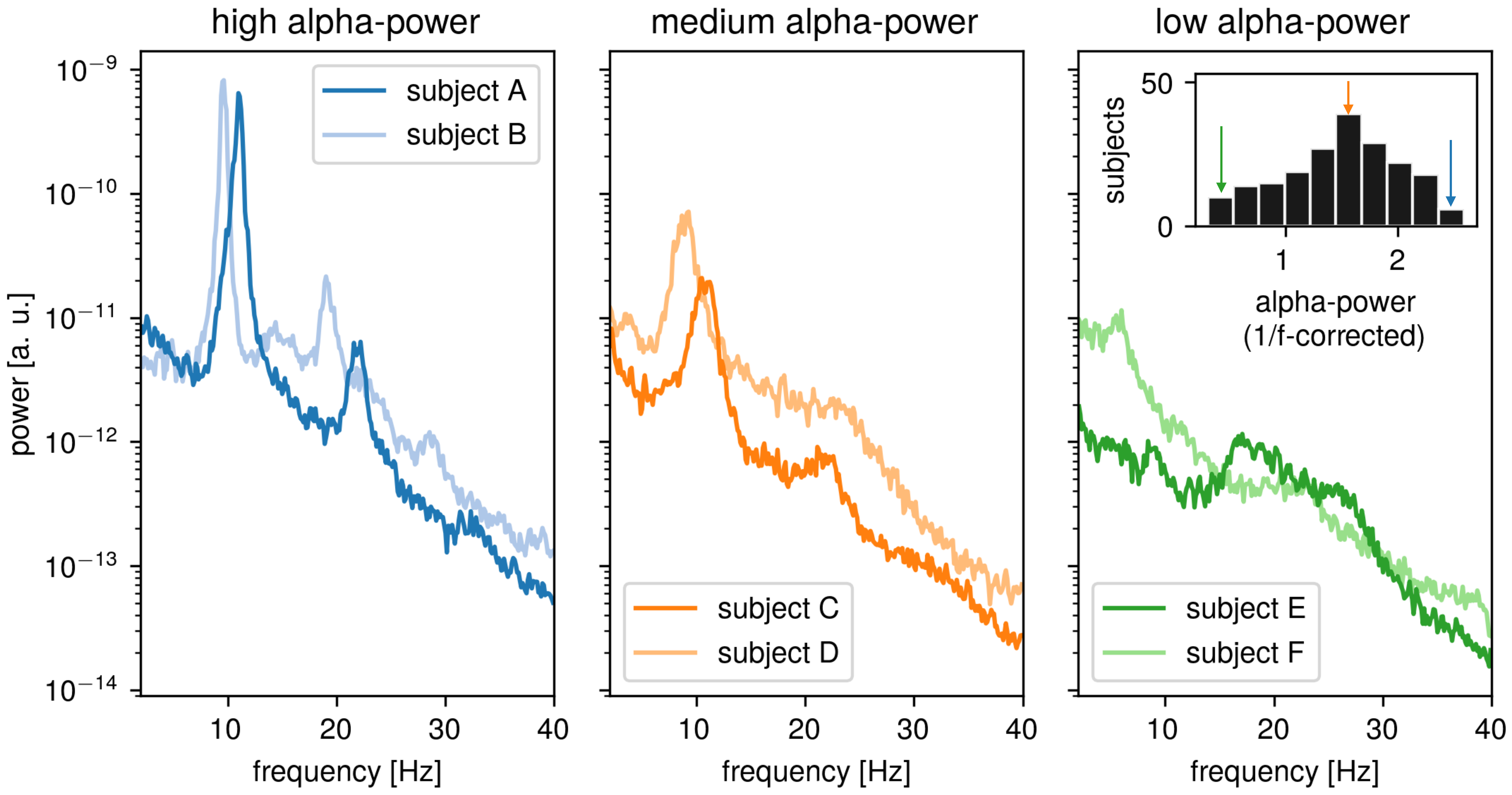




spectral analysis does not explain neuro-physiological generative mechanisms

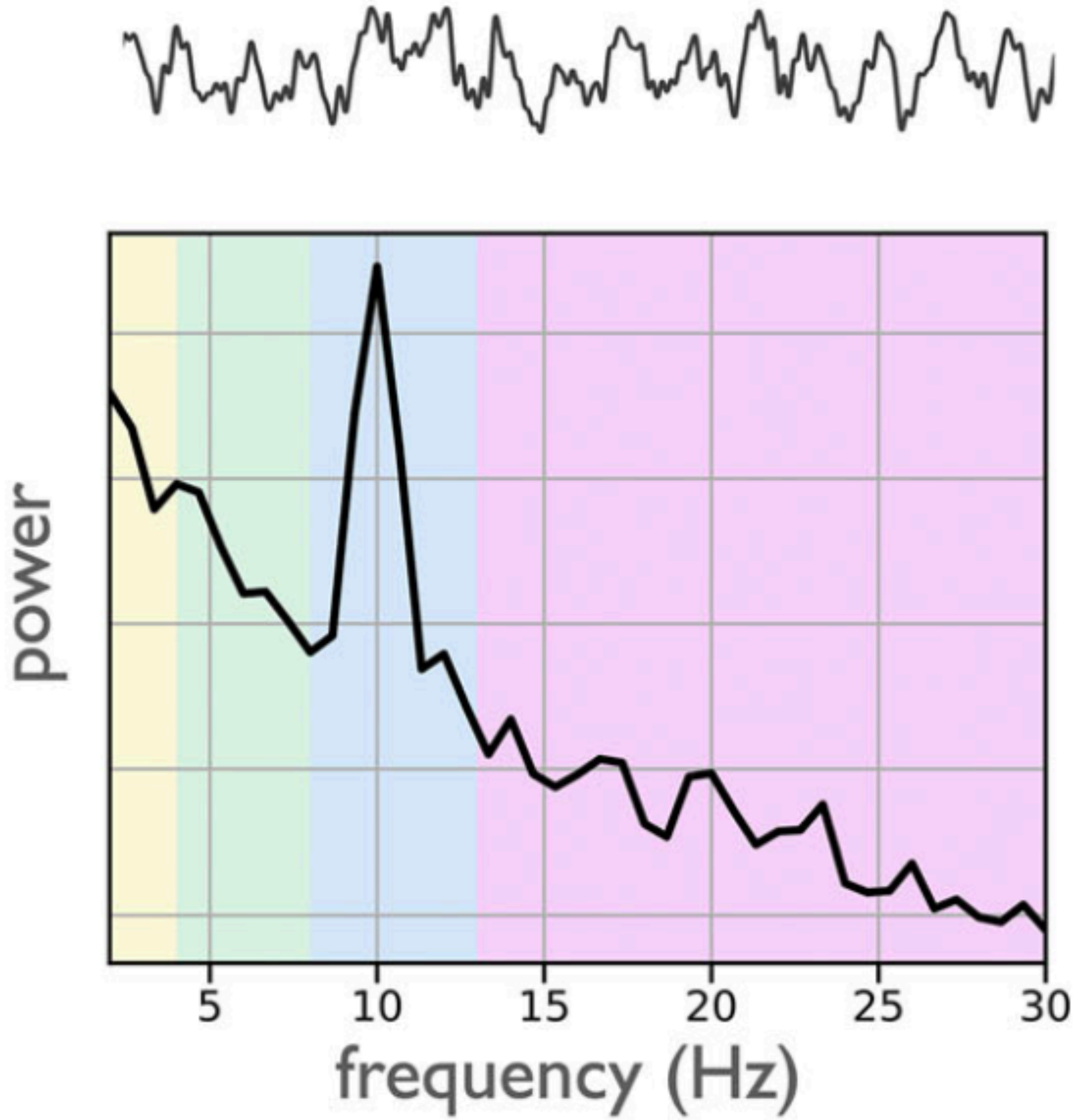
spectral analysis interpretation

variability & oscillation power, example: EEG



Oscillations are variable across participants.

measure: oscillation frequency & power



Frequency (Hz)	IFCN-2017 Glossary
Delta	0.1–<4
Theta	4–<8
Alpha	8–13
Beta	14–30

measure:
"alpha-frequency"

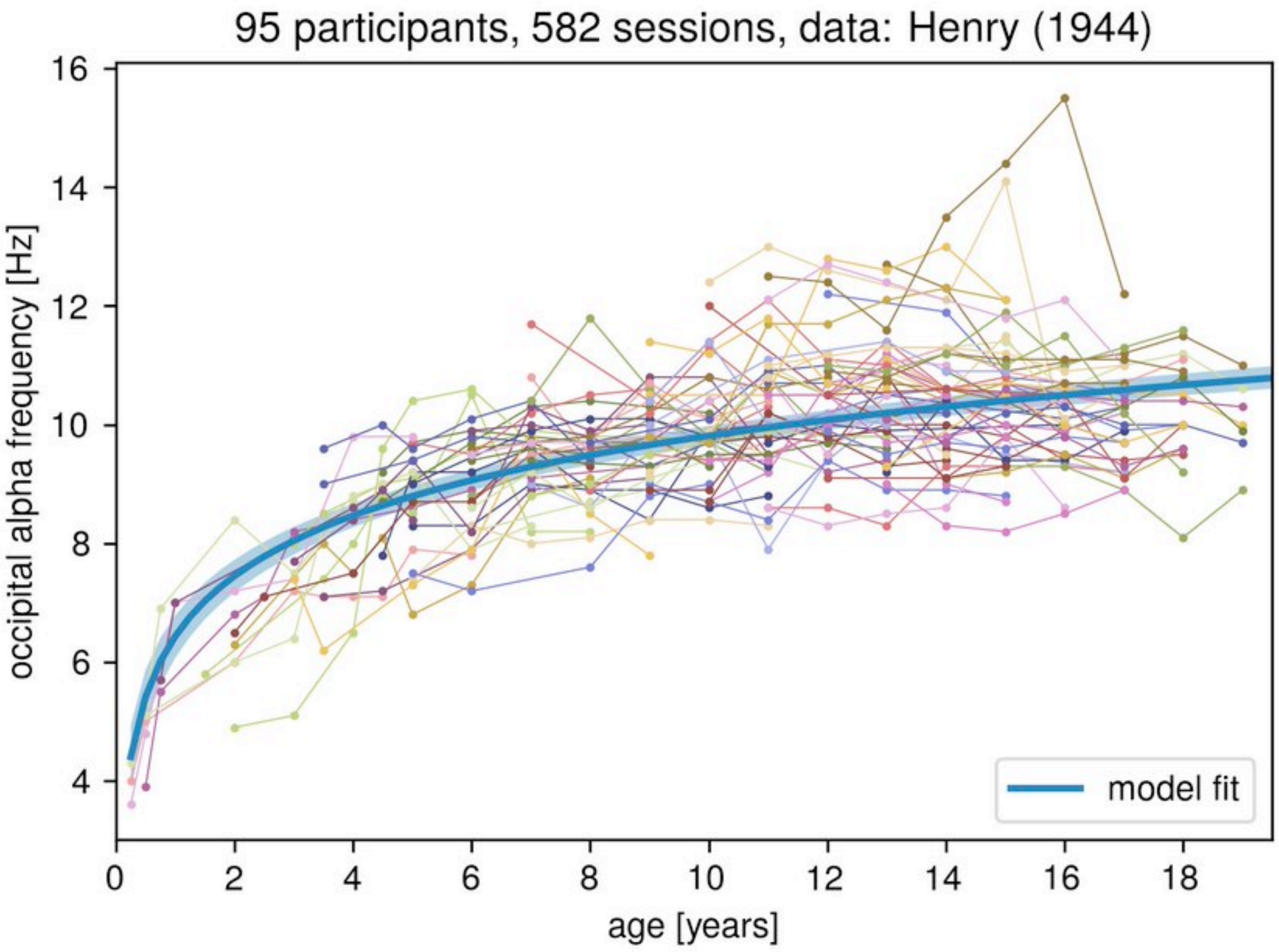
phenomenon:
"alpha-rhythm"

Donoghue, Schaworonkow, Voytek
European Journal of Neuroscience (2021)

pitfalls: frequency changes across frequency boundaries

- observation: oscillation frequency changes drastically over the course of development

- here posterior dominant rhythm, human EEG



pitfalls: frequency changes across frequency boundaries

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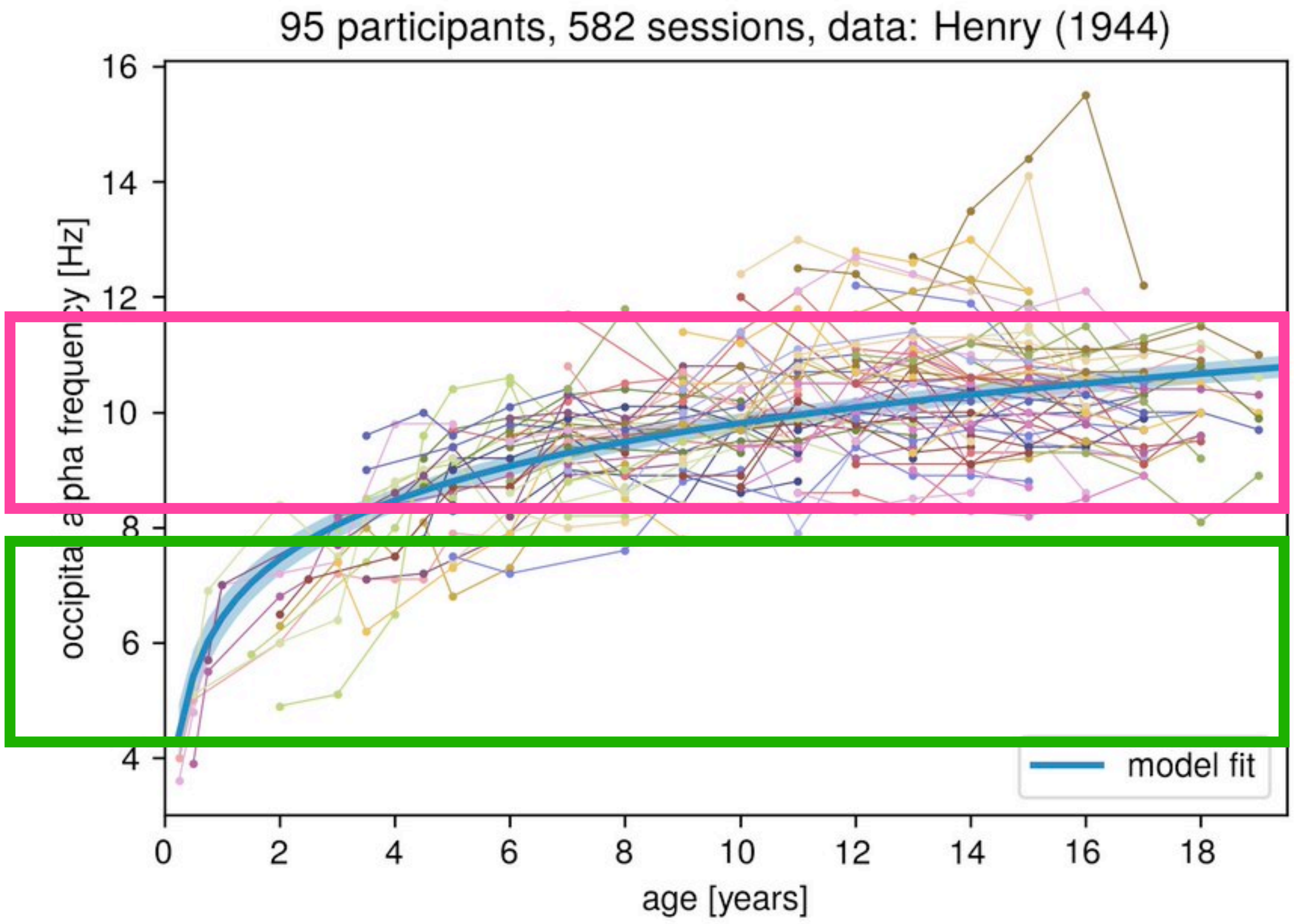
- here posterior dominant rhythm, human EEG

alpha-rhythm?

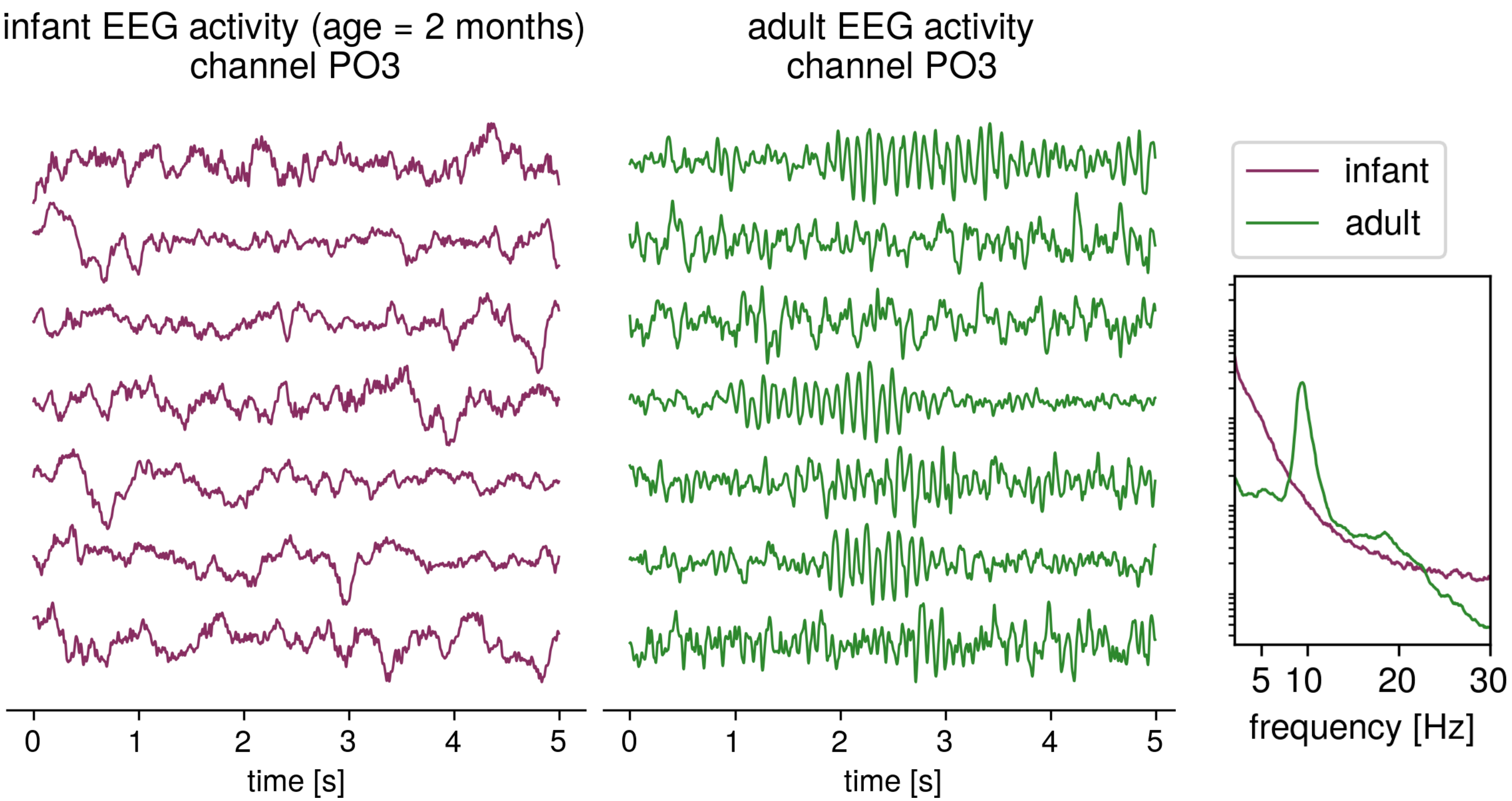
theta-rhythm?



measure: "theta-frequency"

measure: "alpha-frequency"

non-rhythmic contributions, 1/f-activity



Schaworonkow & Voytek
Developmental Cognitive Neuroscience (2021)

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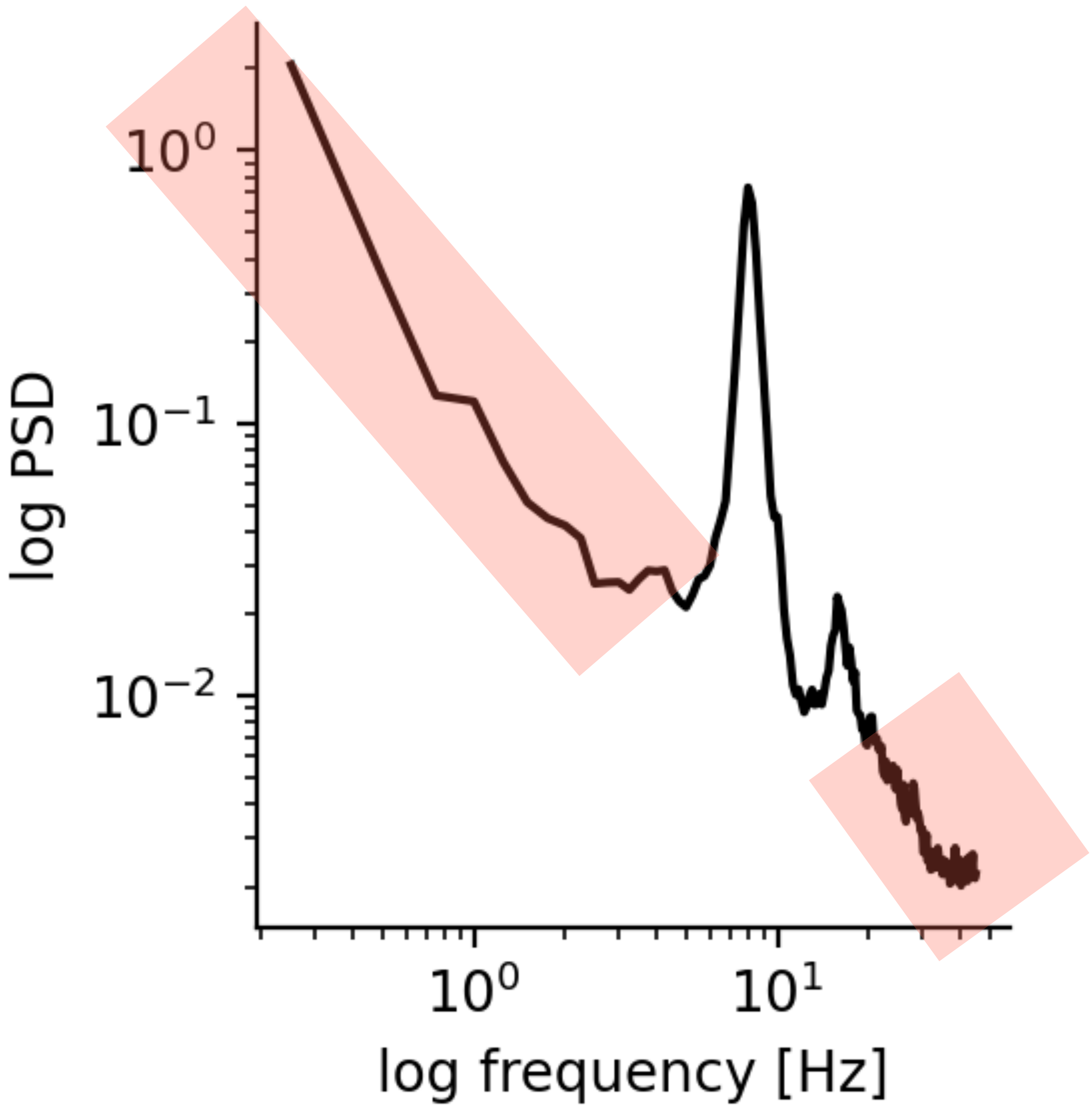
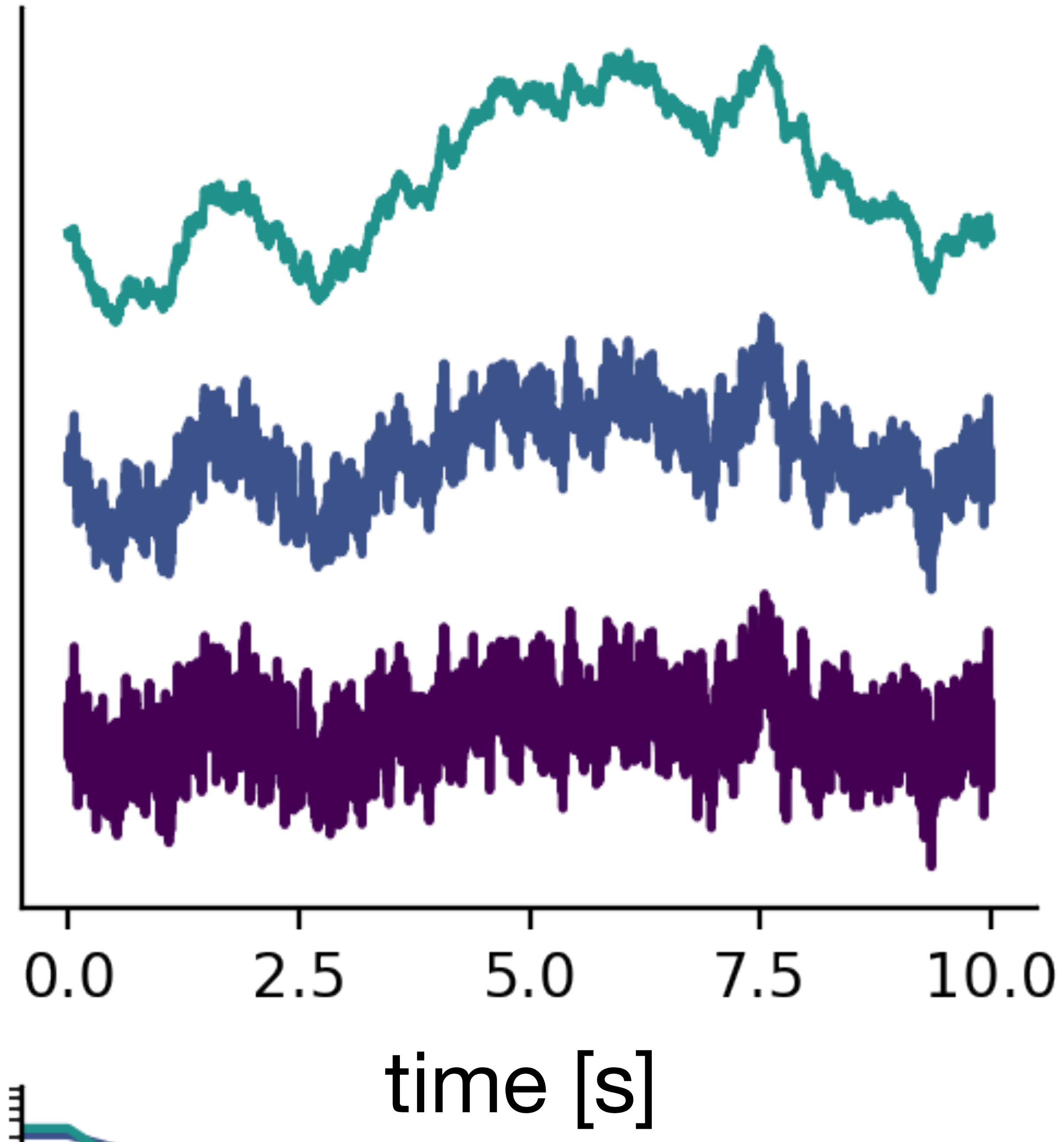
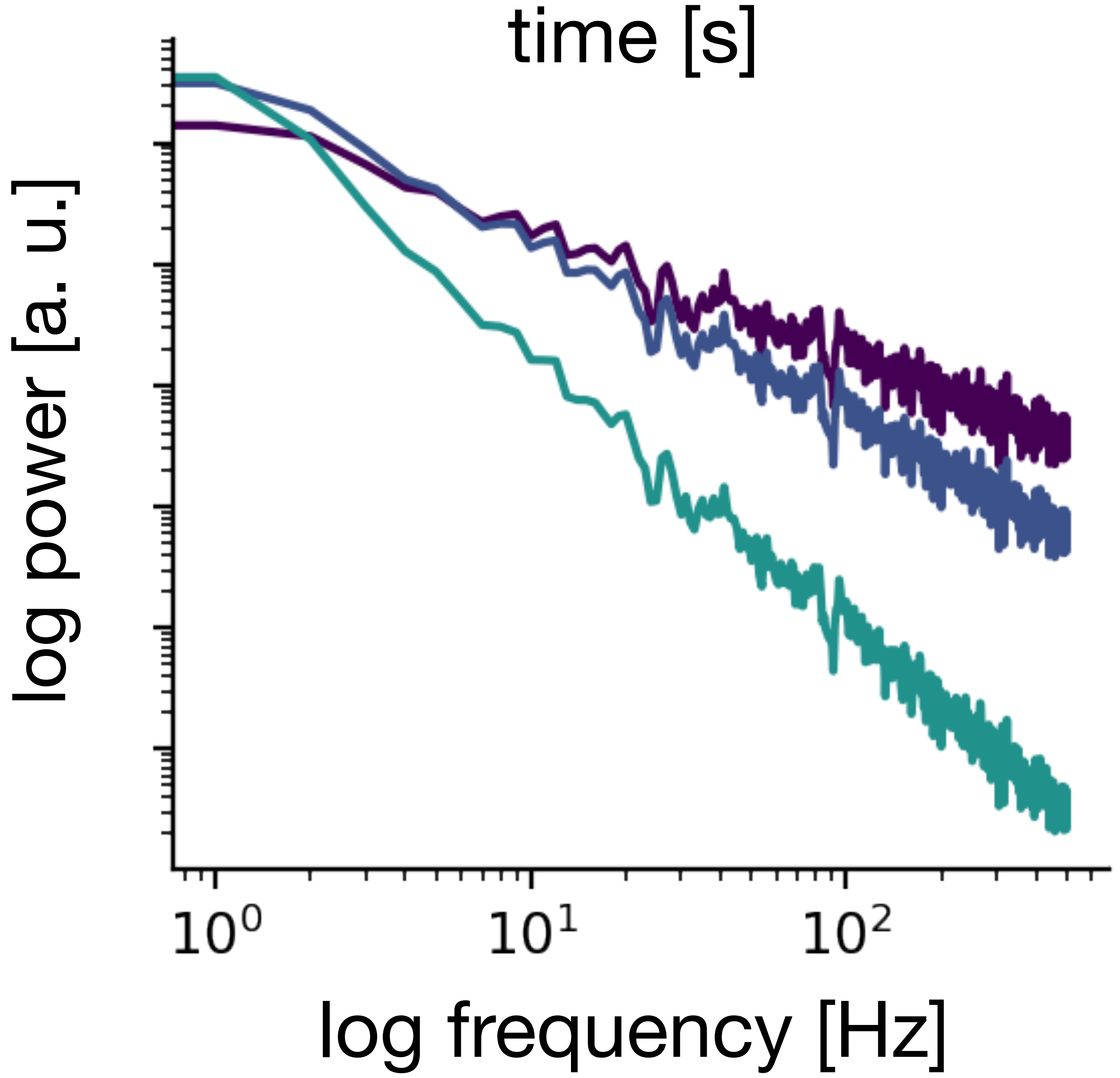
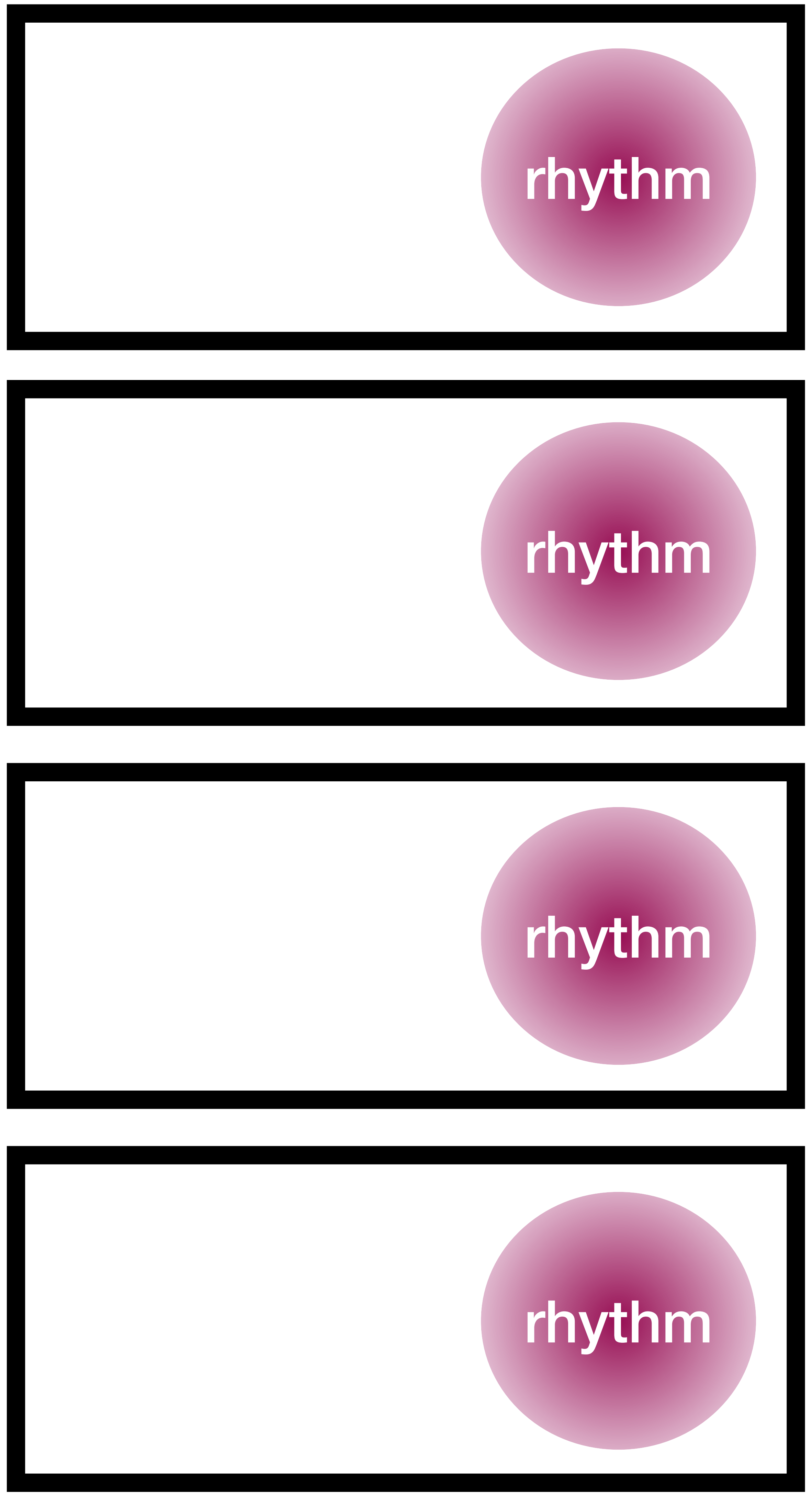






- 1/f-activity:
  power scales with frequency
  scale free dynamics

pitfalls: not only rhythmic changes + rhythms can be absent



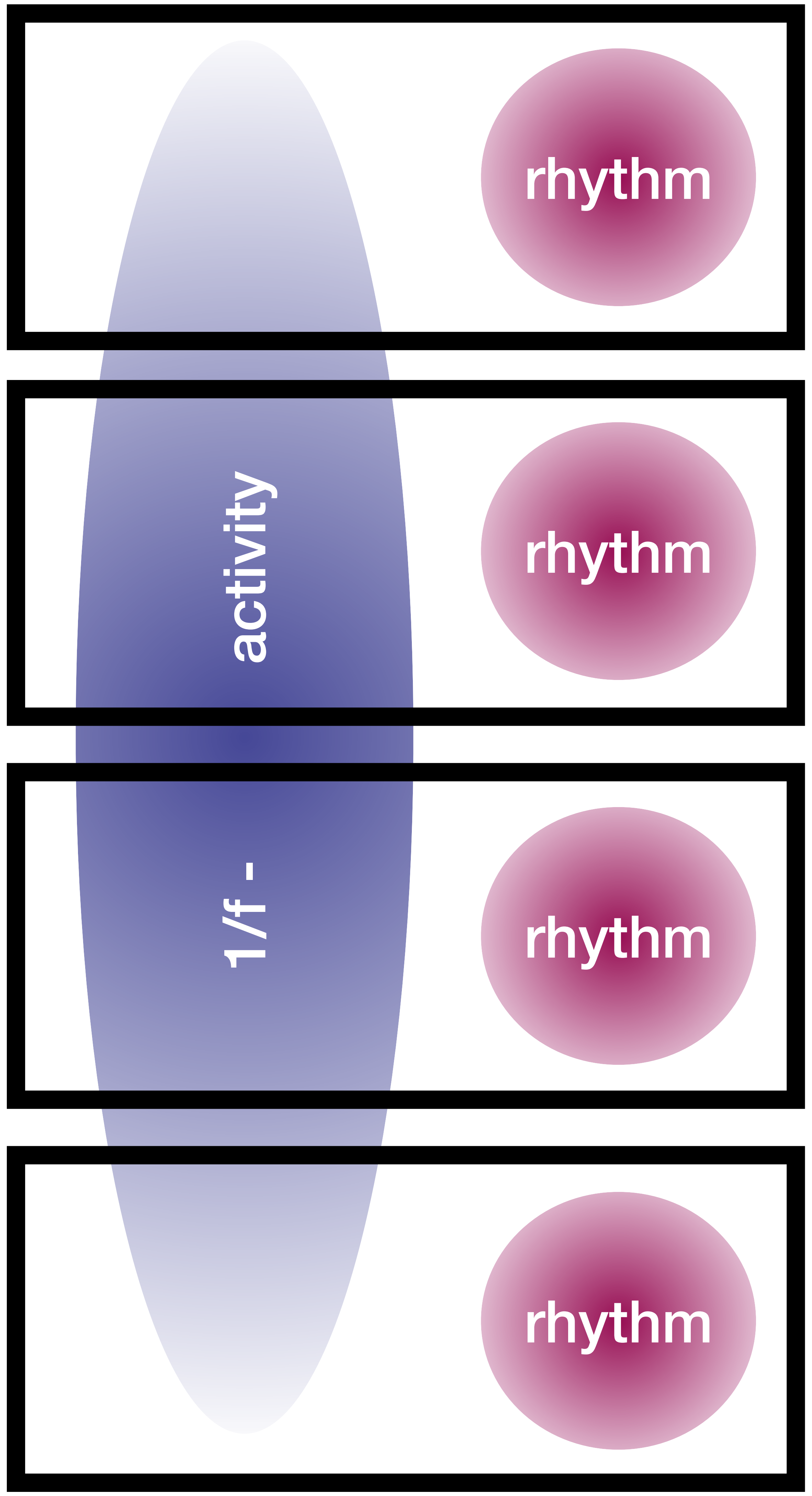
“delta-power”

“theta-power”

“alpha-power”

“beta-power”

pitfalls: not only rhythmic changes + rhythms can be absent



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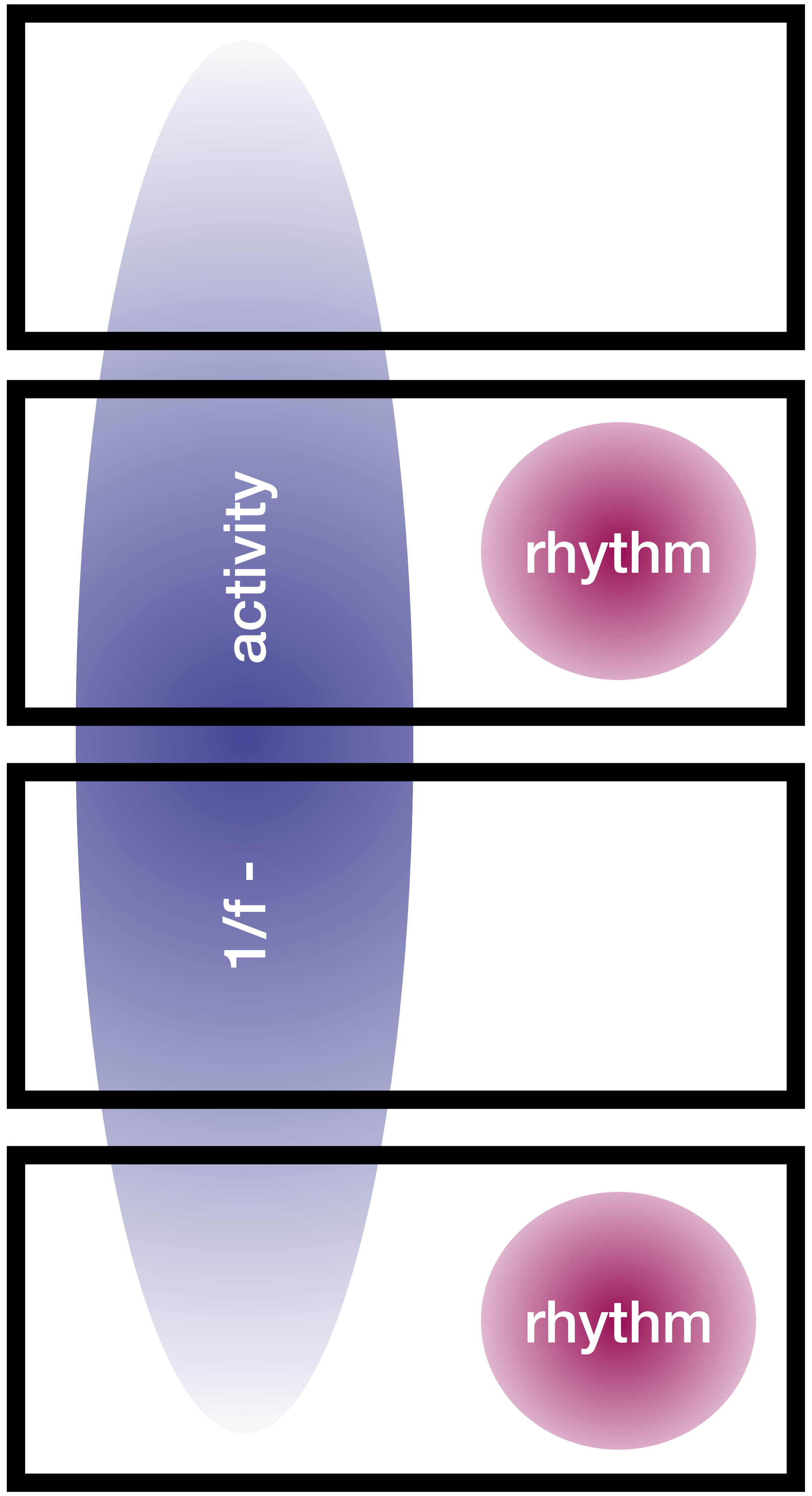

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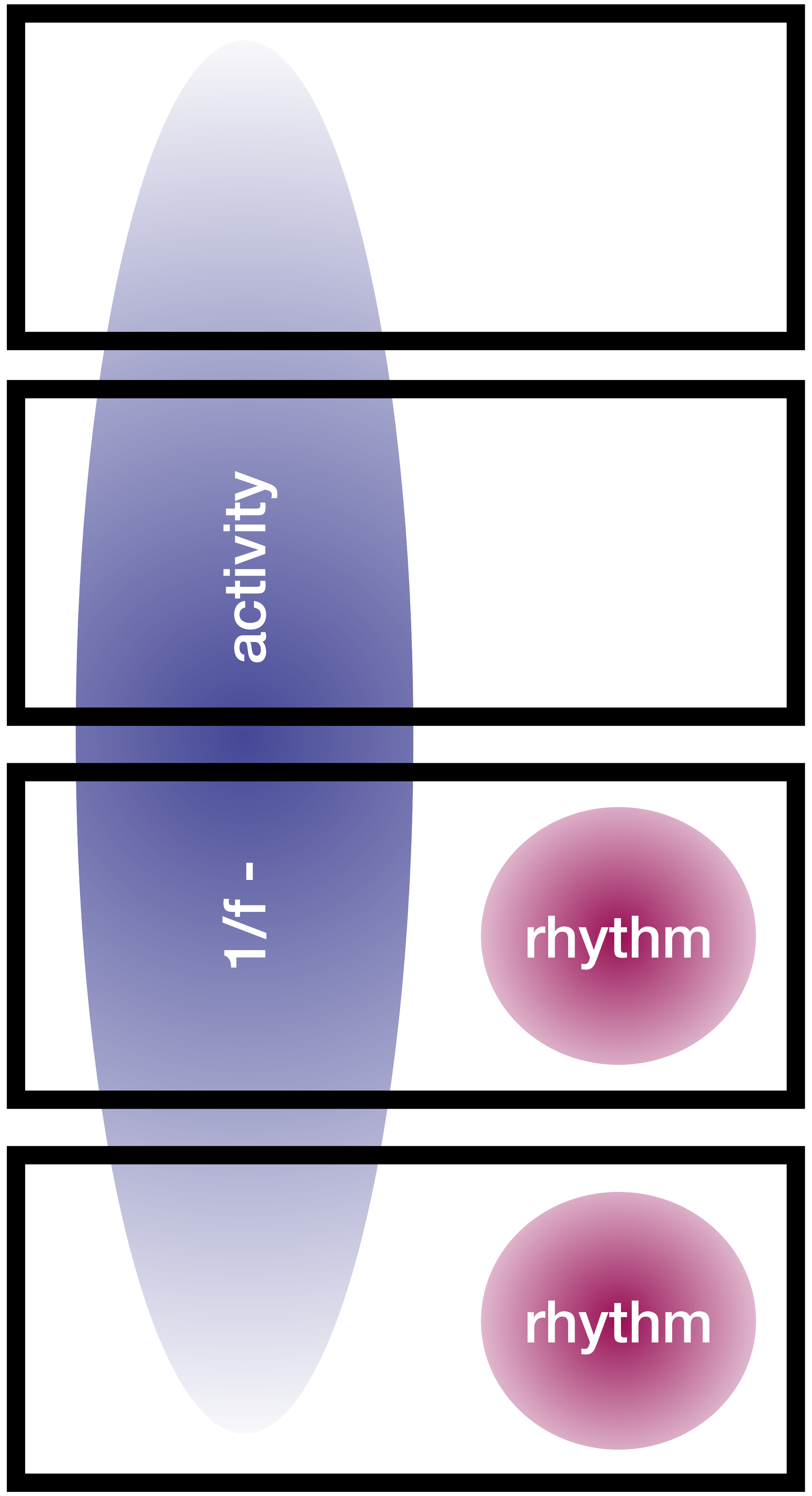

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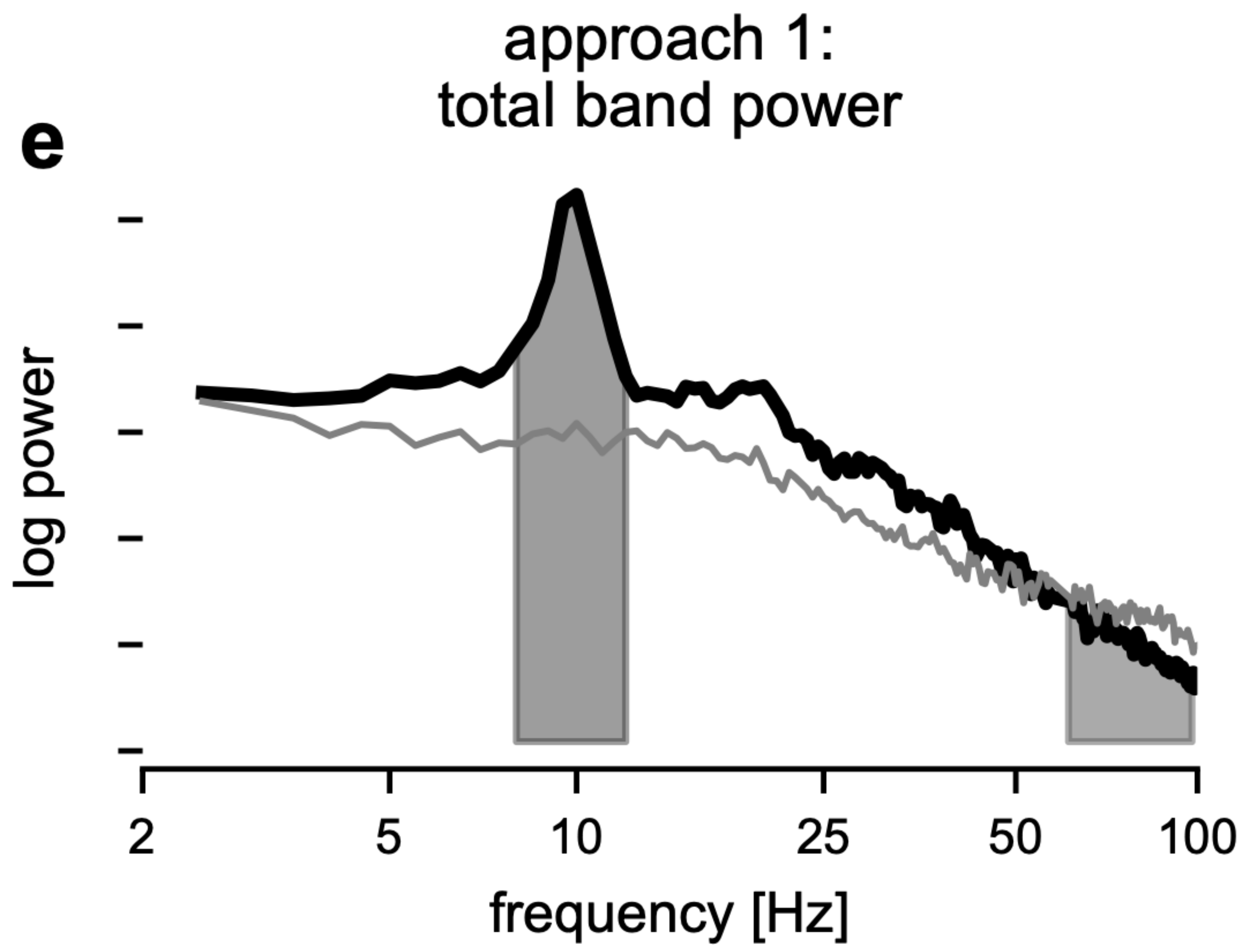
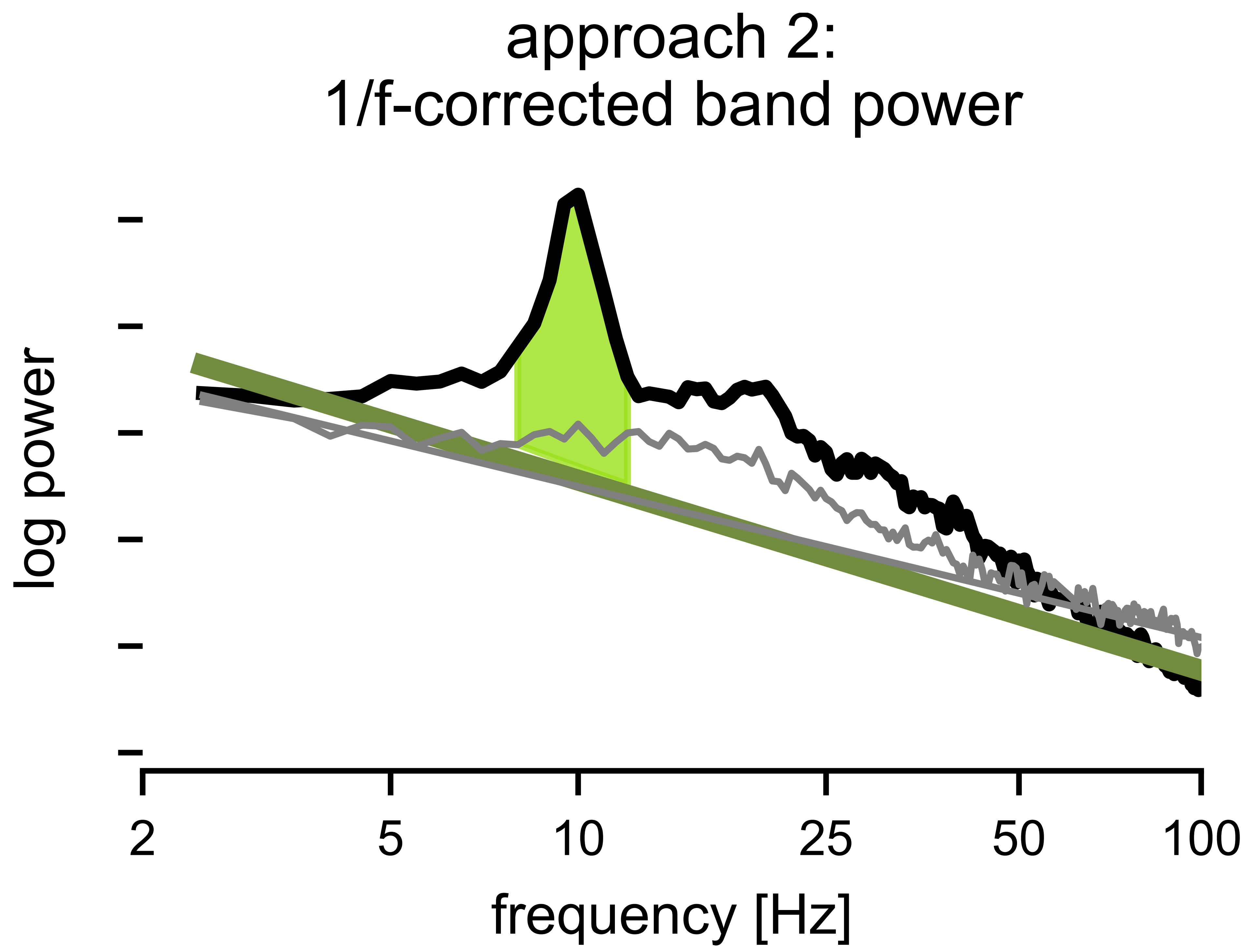
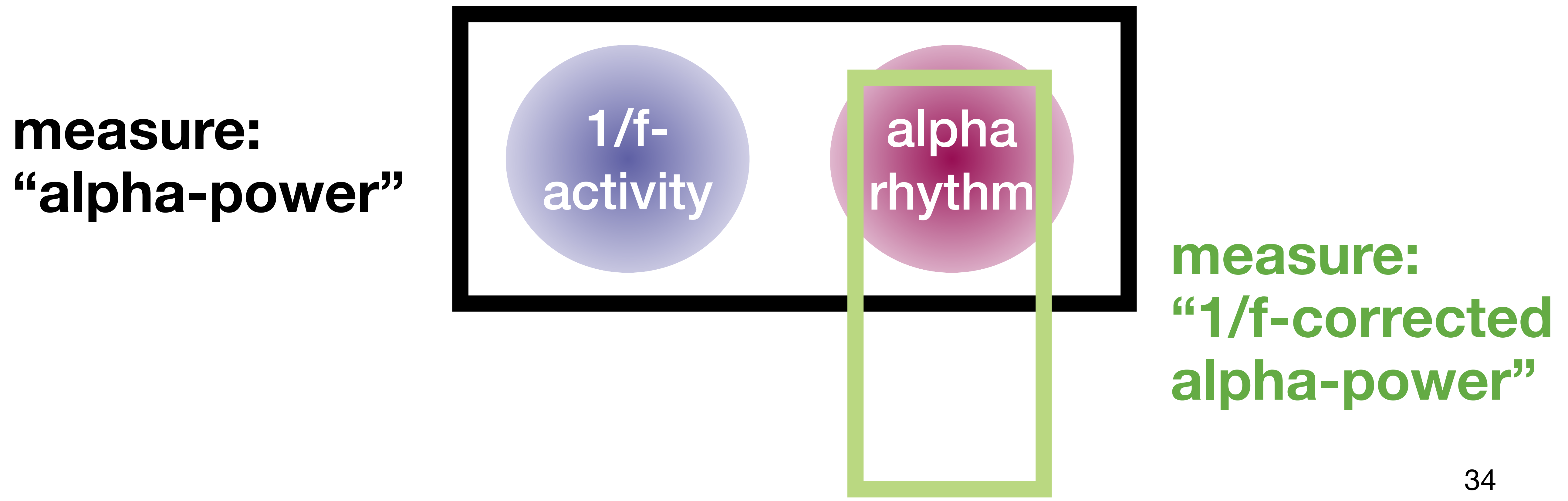
new measures needed?





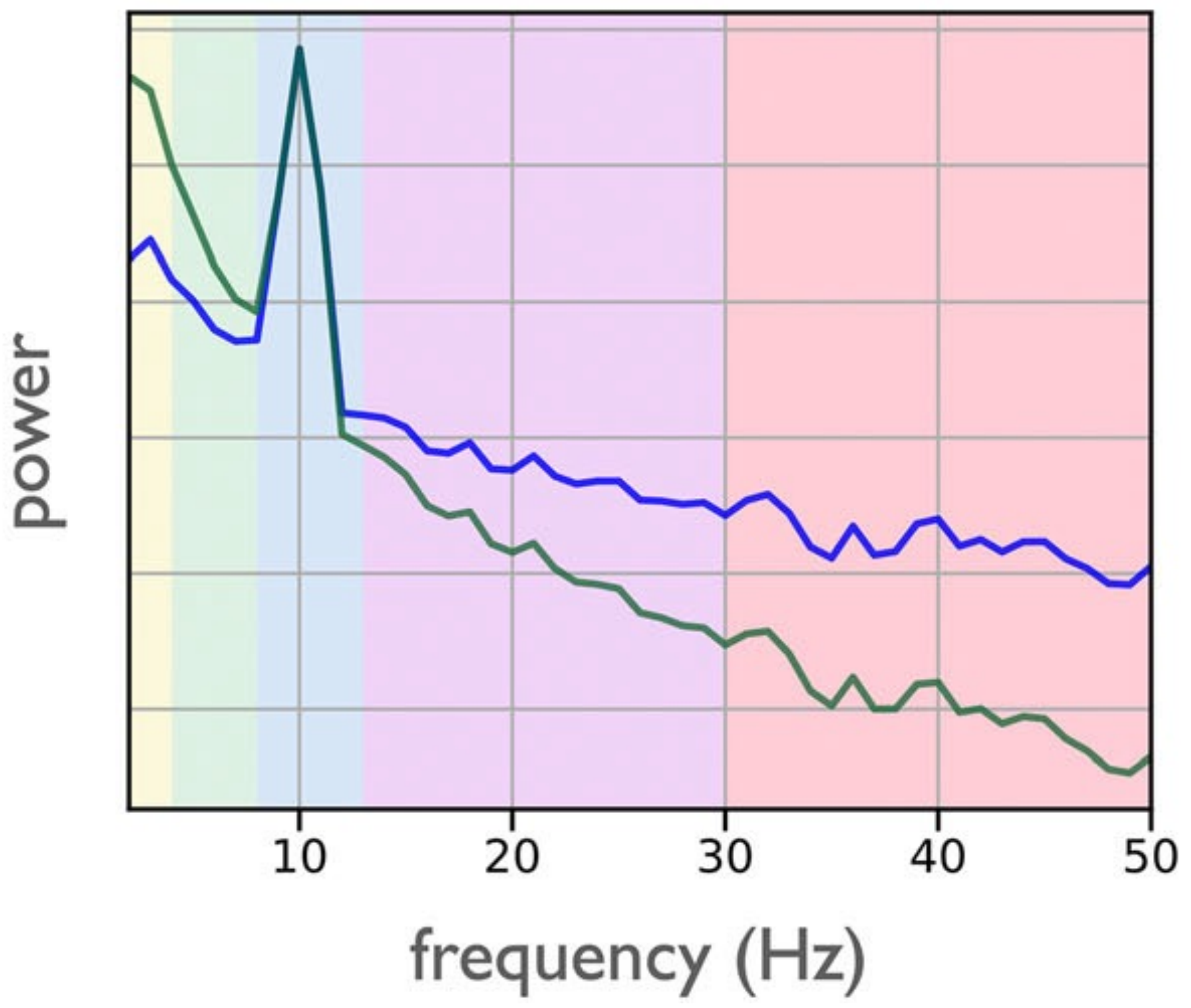


1/f-corrected alpha power = improved way to measure oscillatory contribution more specifically?

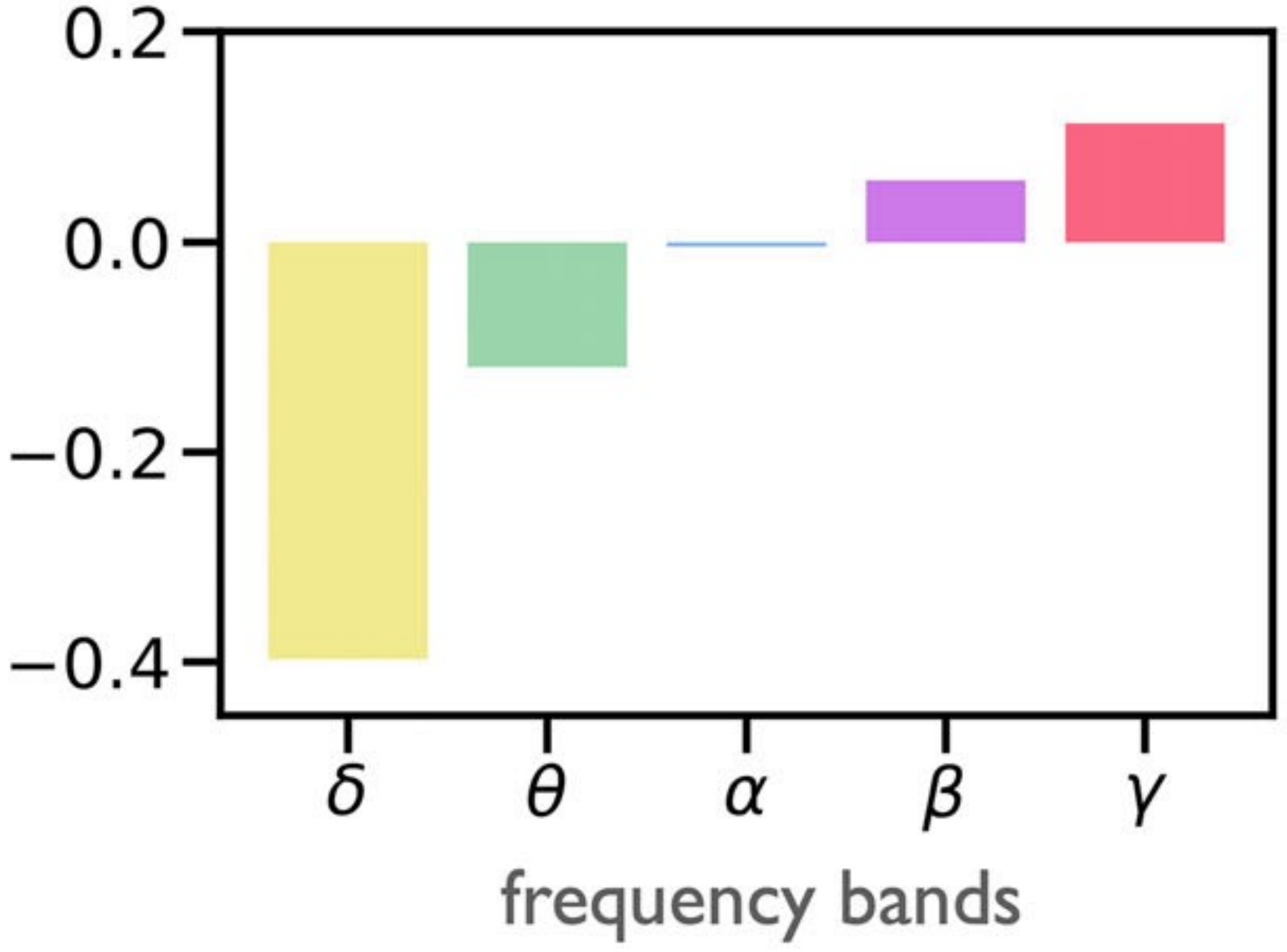
new measures needed?

Two ways to plot the same result:

changes between 2 conditions in terms of 1/f-exponent



changes between 2 conditions per frequency band



Donoghue, Schaworonkow, Voytek
European Journal of Neuroscience, 2021

same summary measures, different signal generation

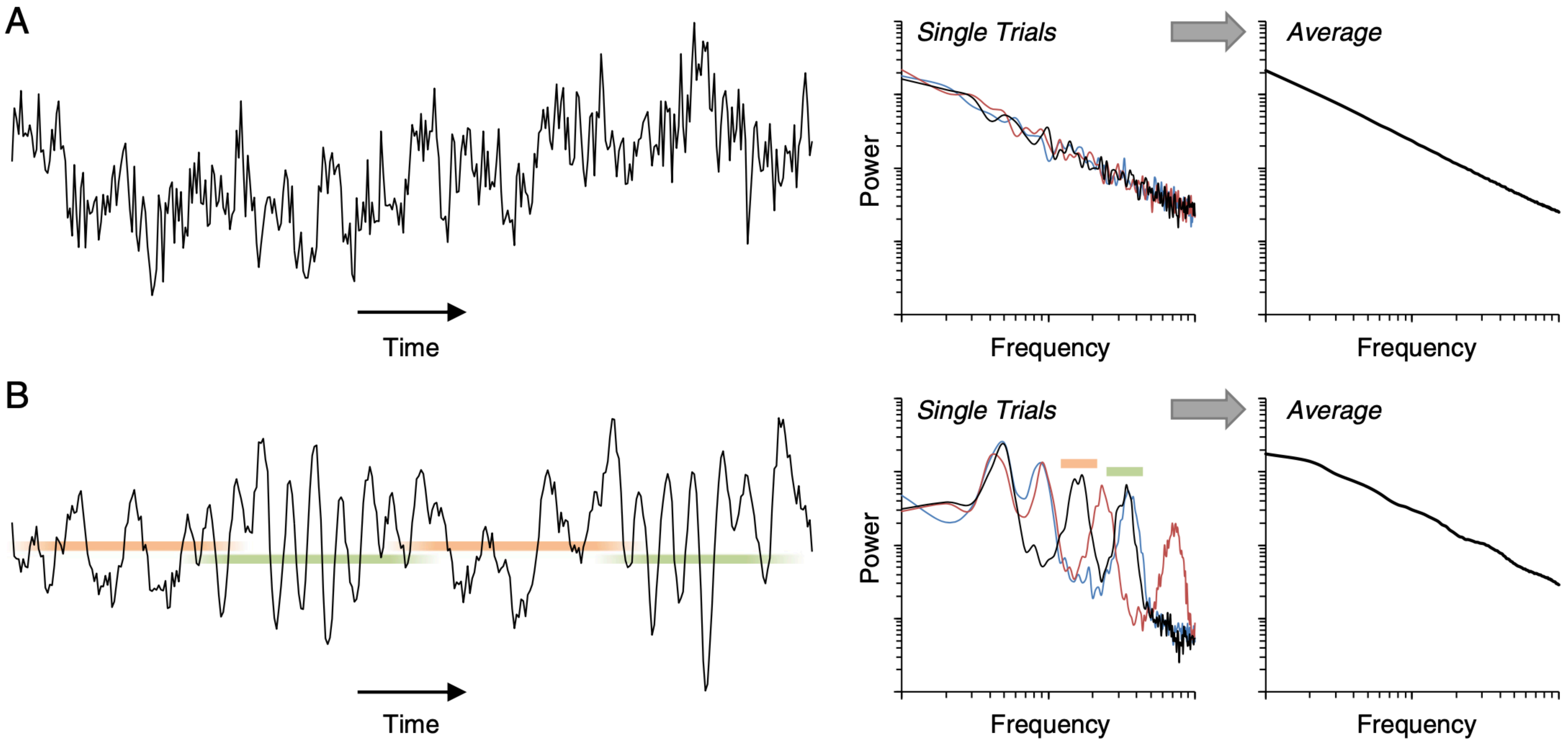
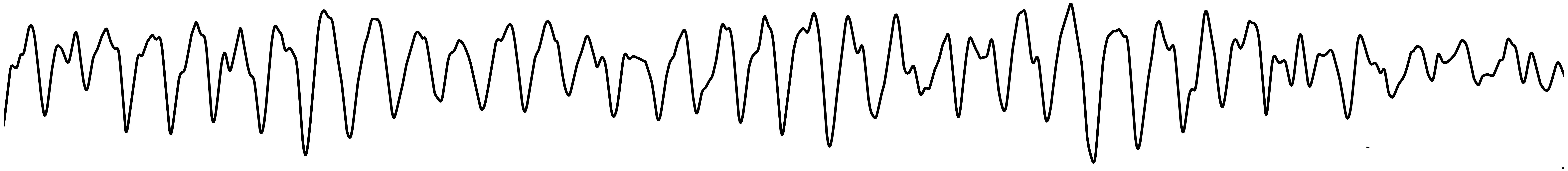
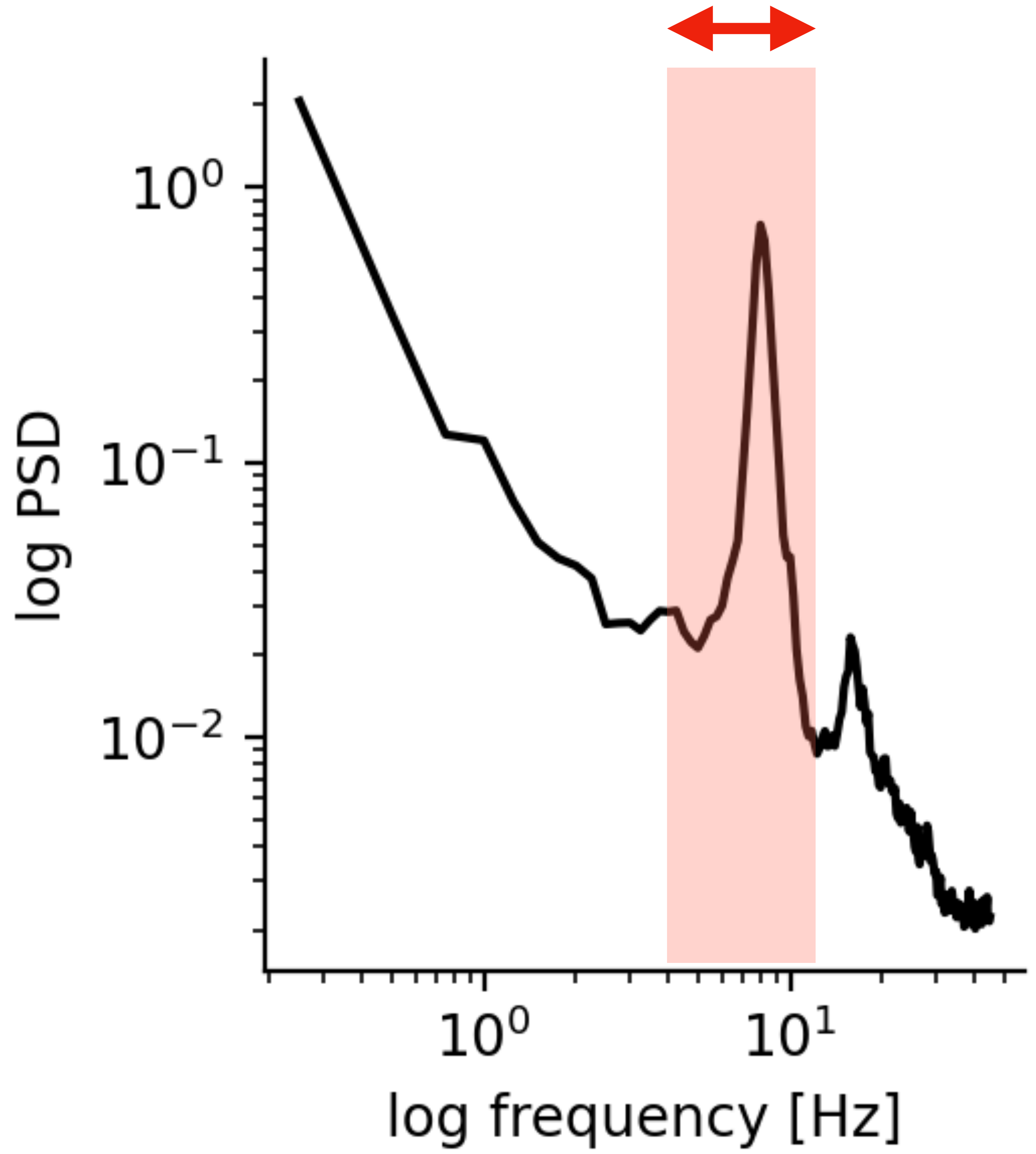


figure from: Palva & Palva
NeuroImage, 2012

phenomenon: amplitude modulation

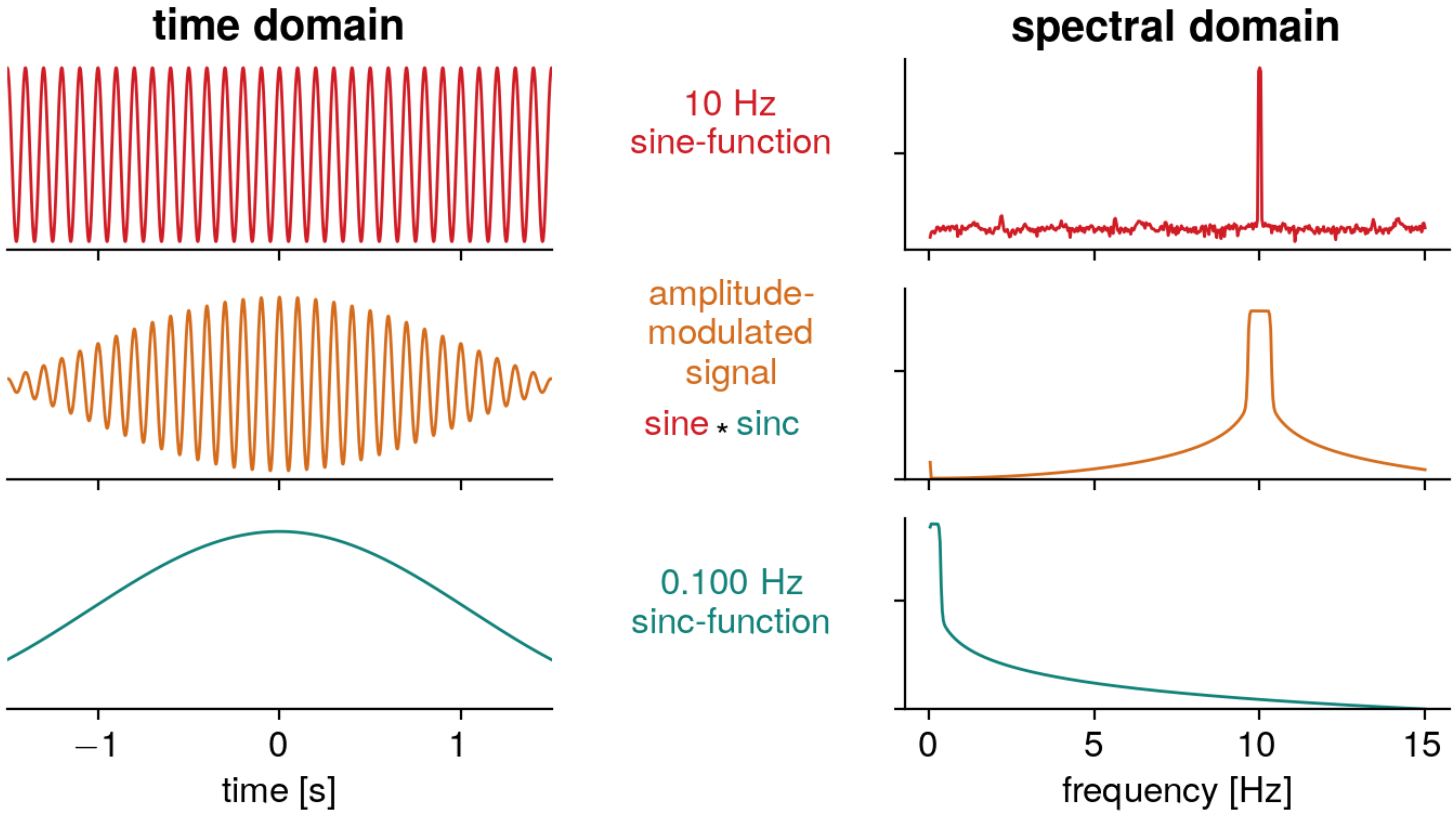
- neural rhythms show amplitude modulation.





effects of amplitude modulation on spectrum

- amplitude modulation can contribute to a broadened spectral peak.



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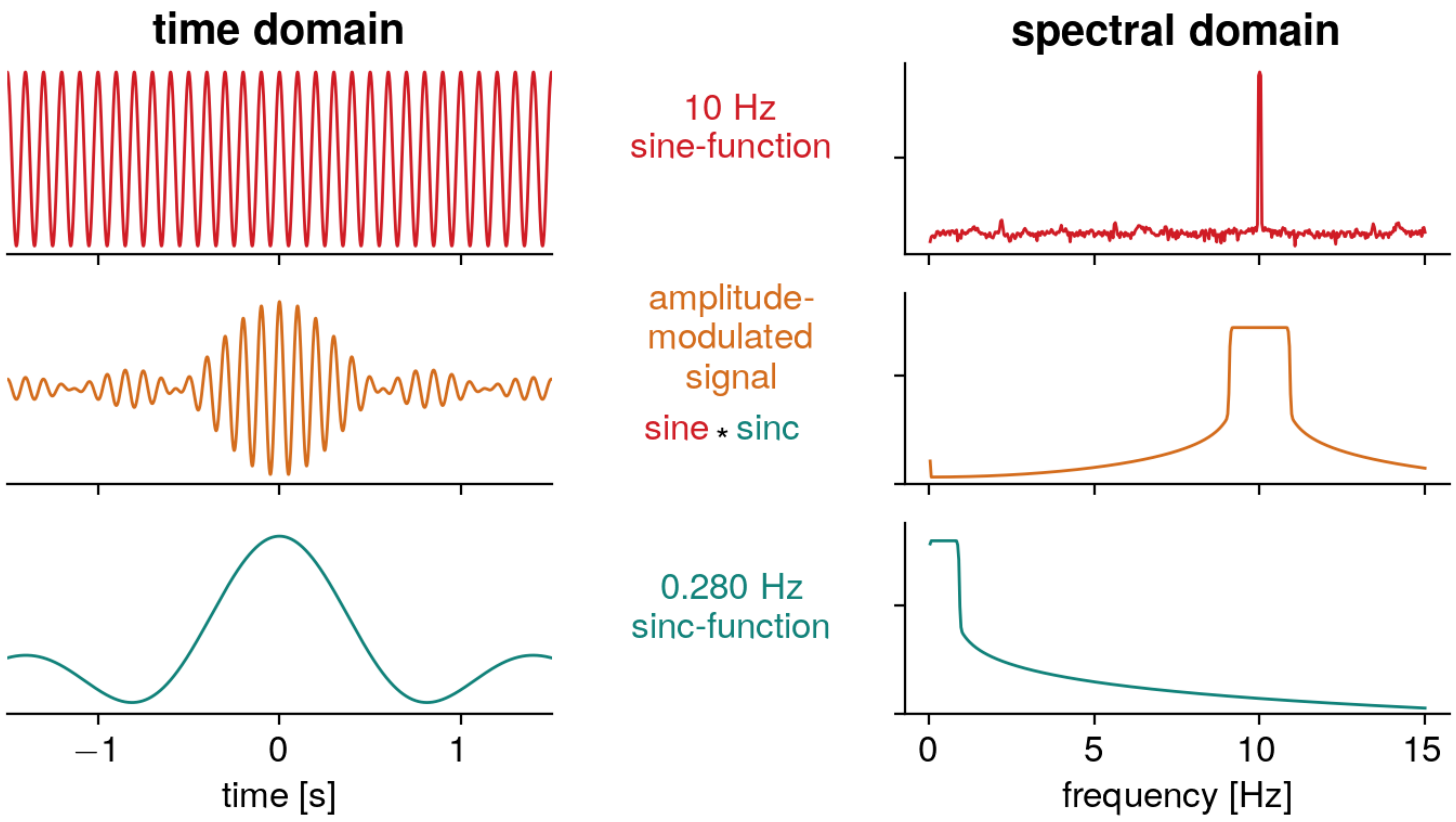
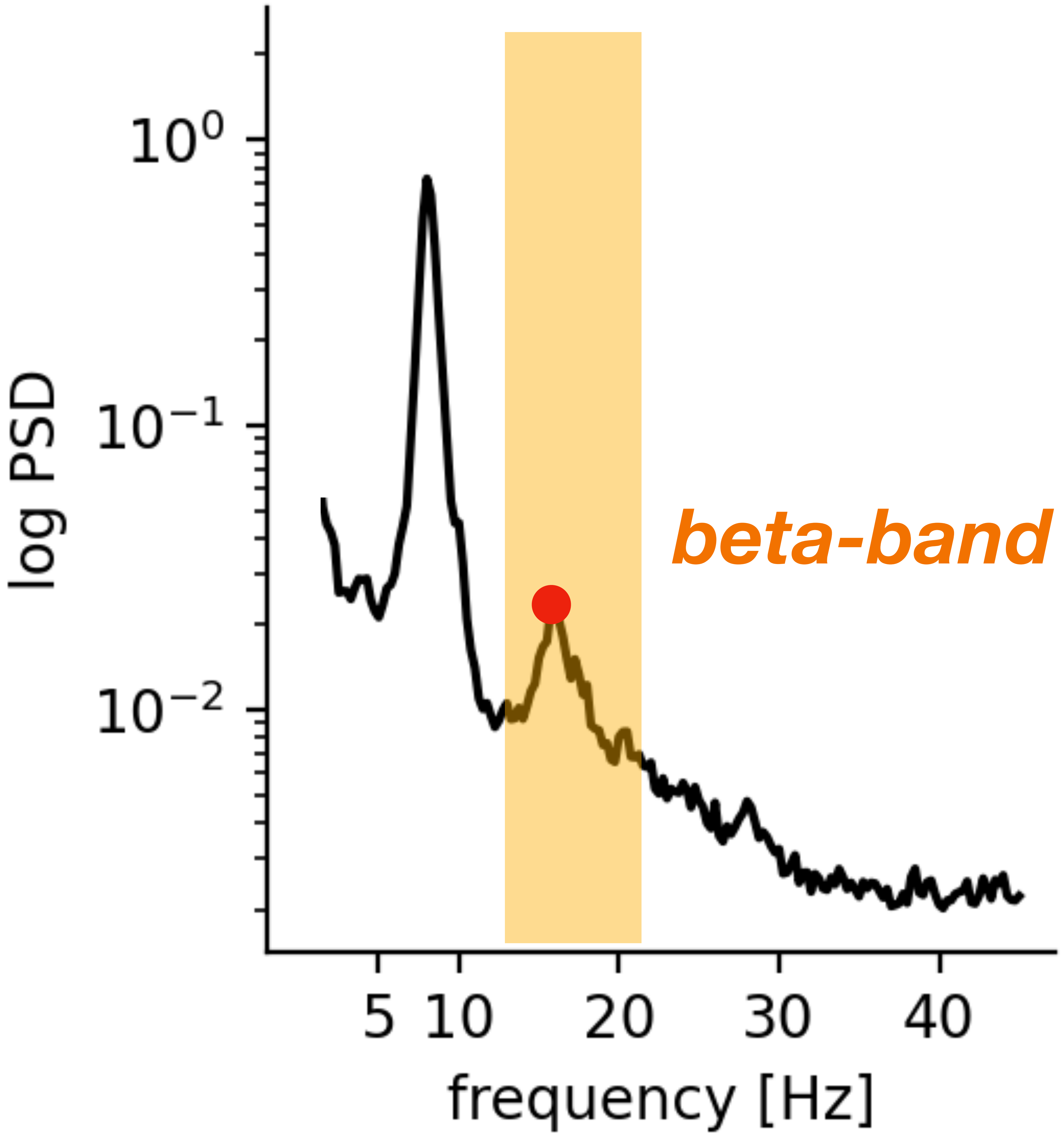


higher frequency spectral peaks

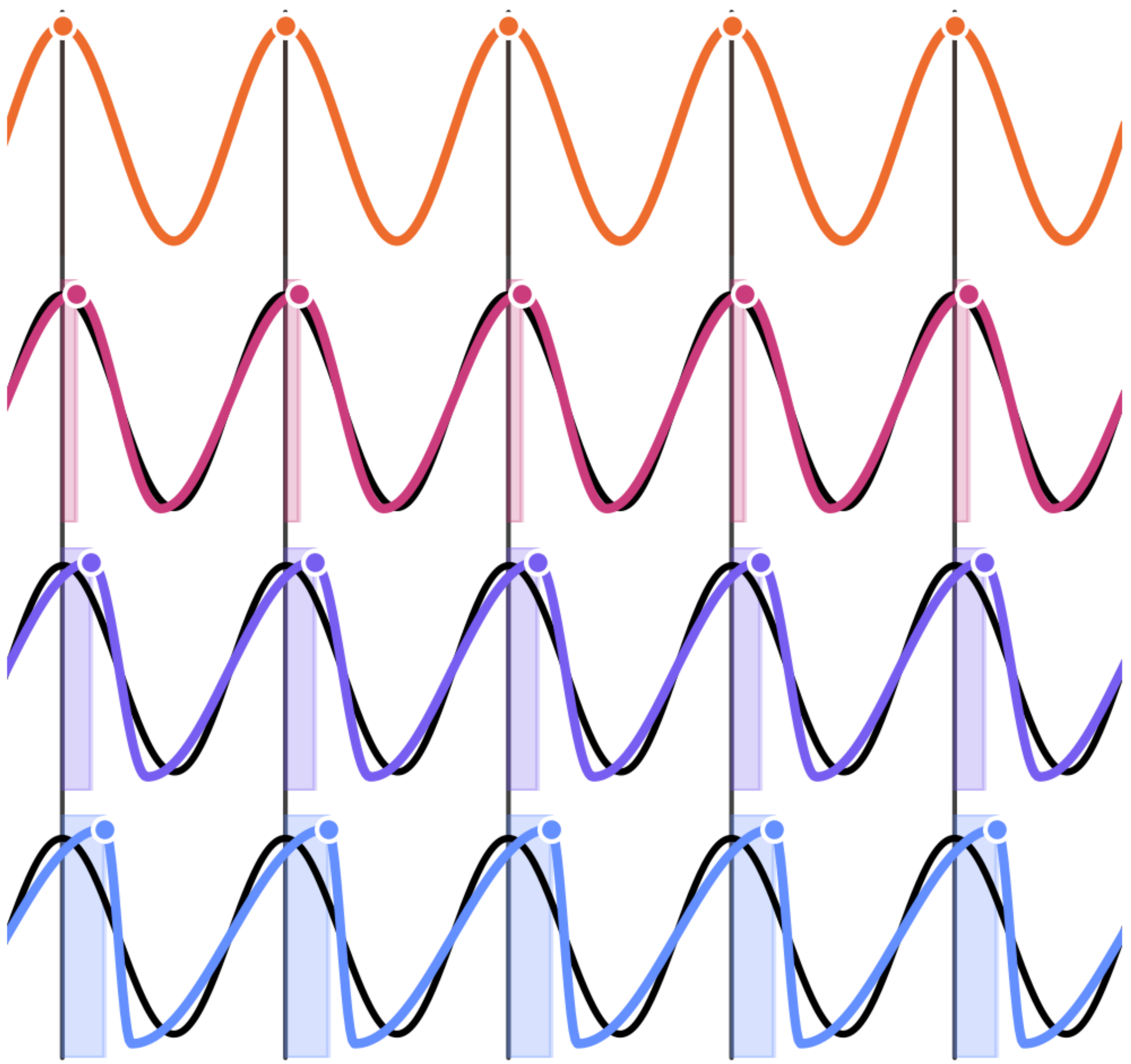


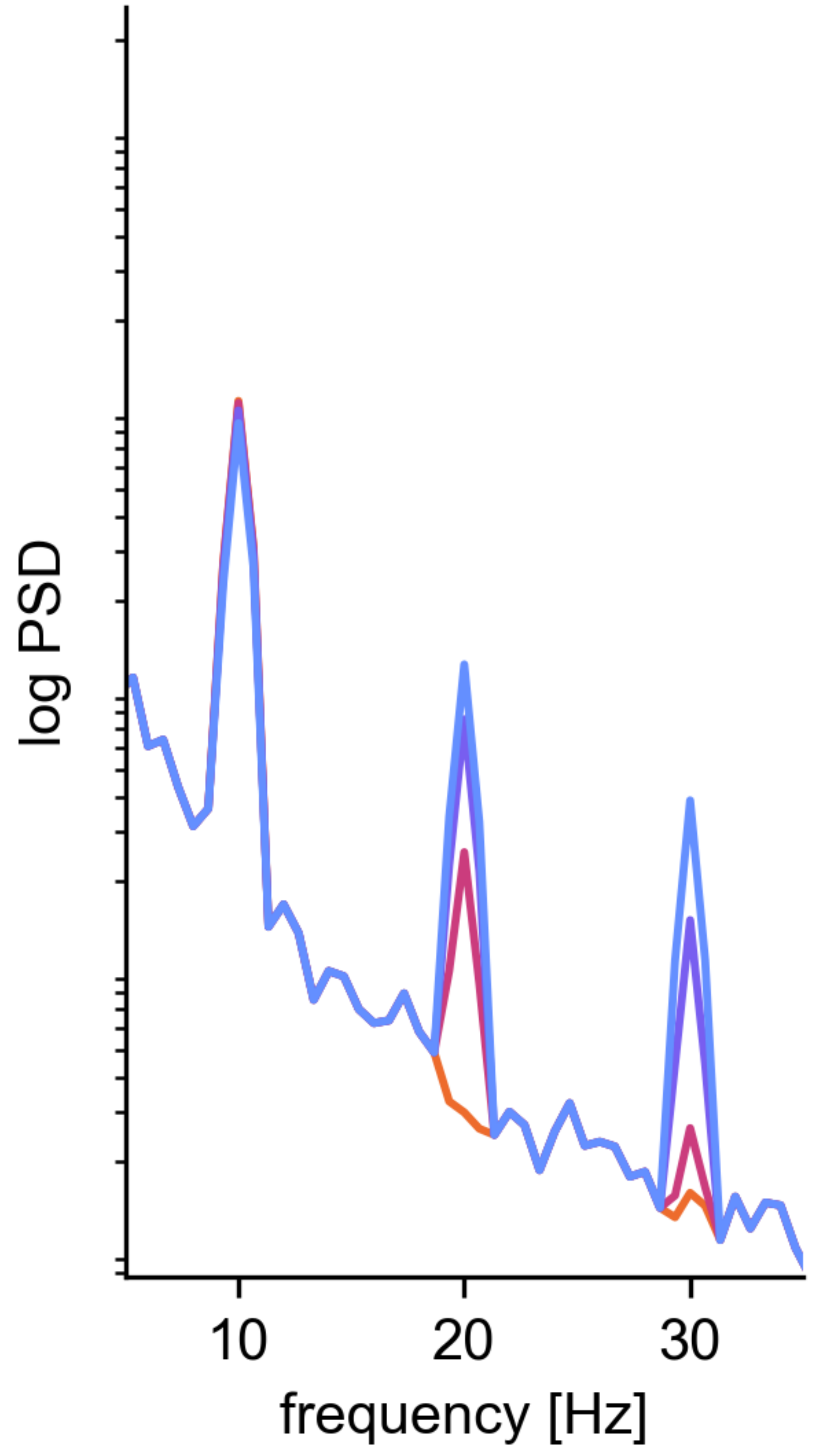
✅ verify presence of peak

✅ identify frequency of peak in beta-band

- additional spectral peaks in higher frequency bands: genuine rhythms or harmonic contributions?

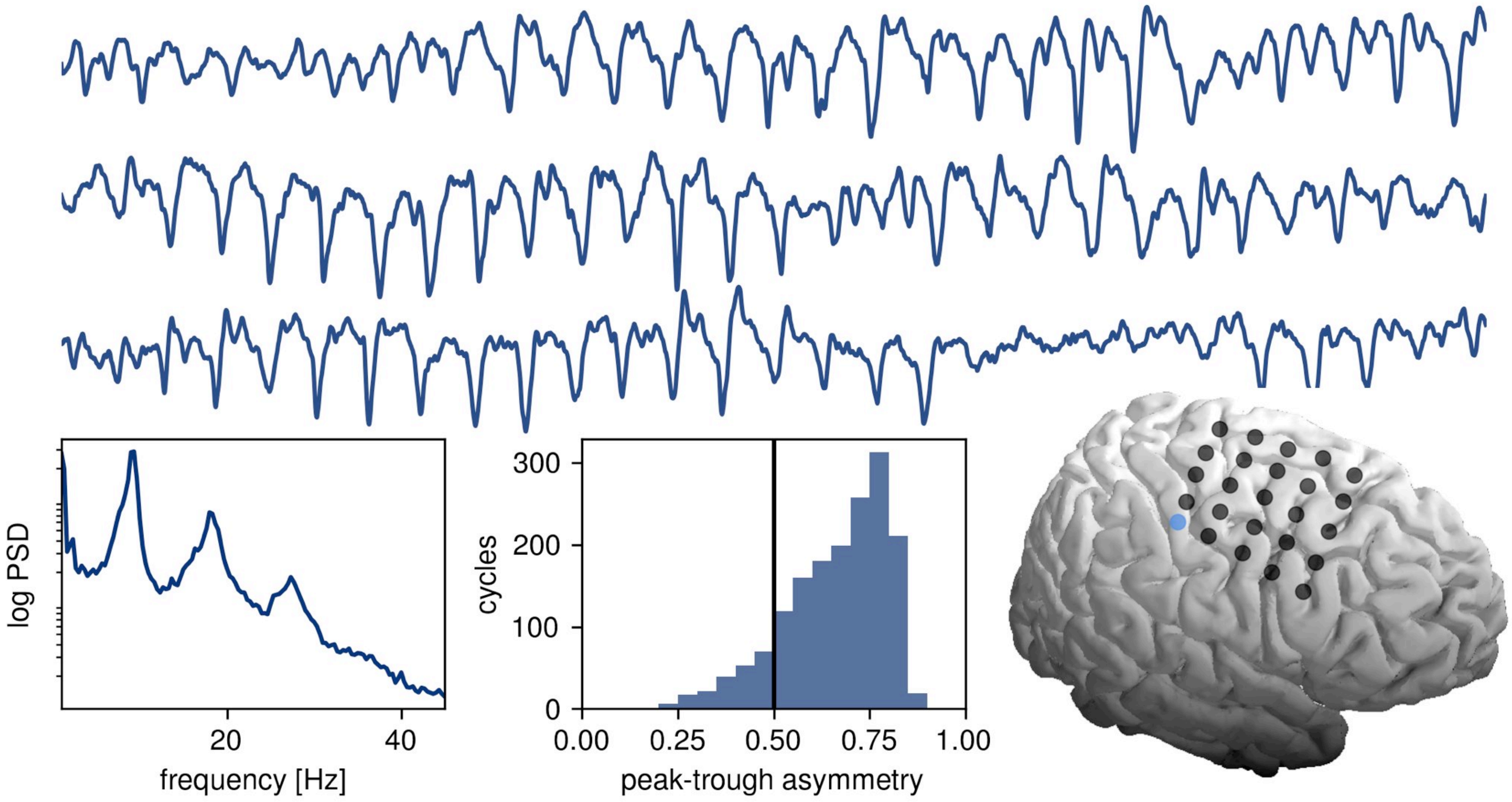
pitfalls: waveform shape & spectral peaks



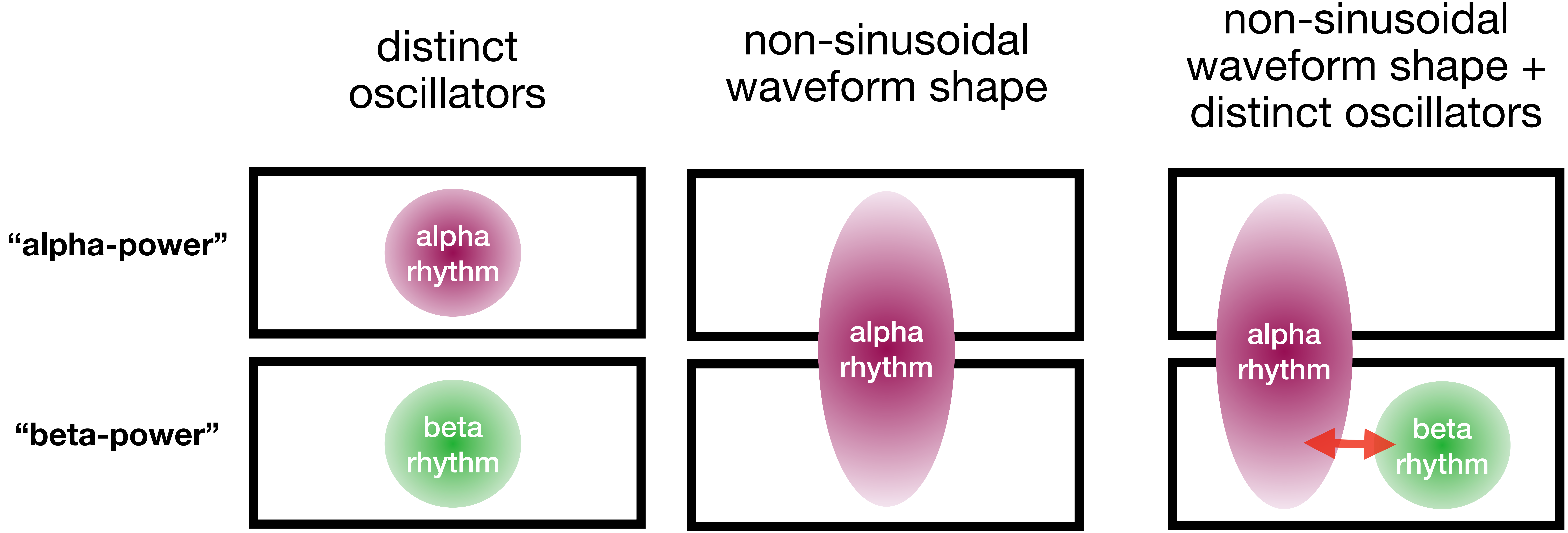


- individual spectral peaks ≠ independent rhythms
- non-sinusoidal waveforms → harmonic peaks in the spectrum.

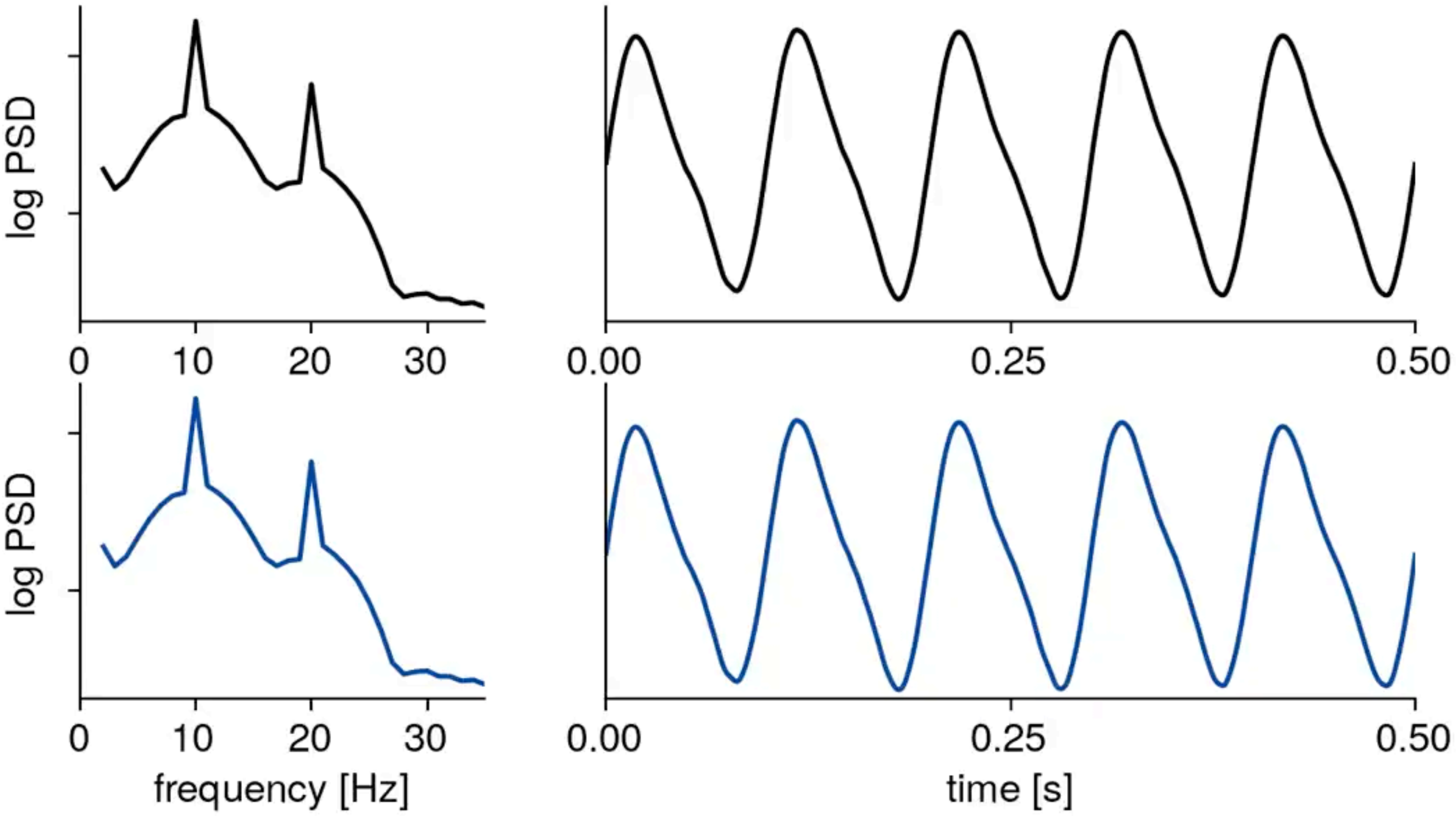
example: non-sinusoidal waveform in ECoG data



waveform shape & spectral peaks: concepts

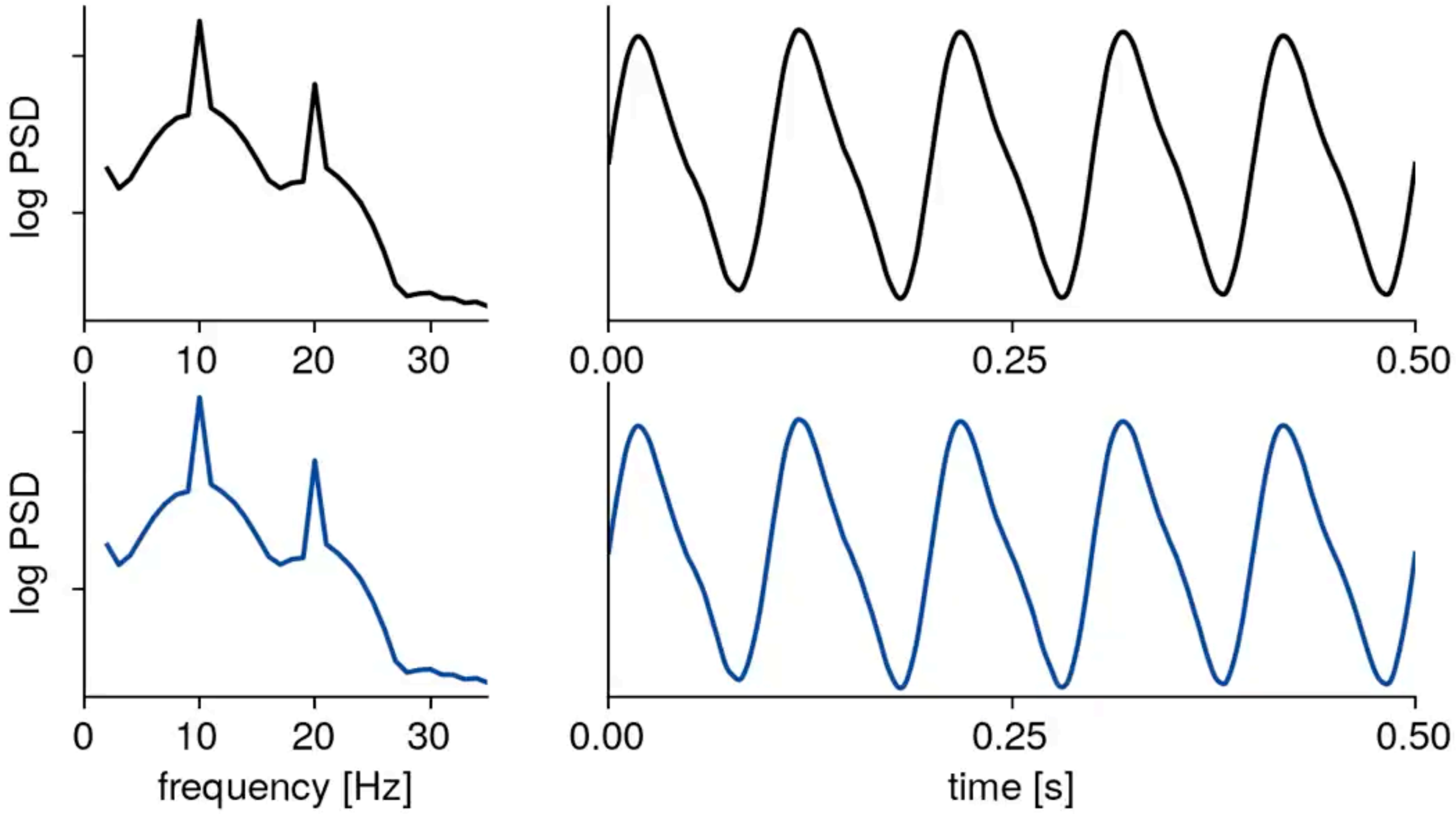


what about phase?



same spectrum – different time domain signals: the power spectrum only captures a part of the available information

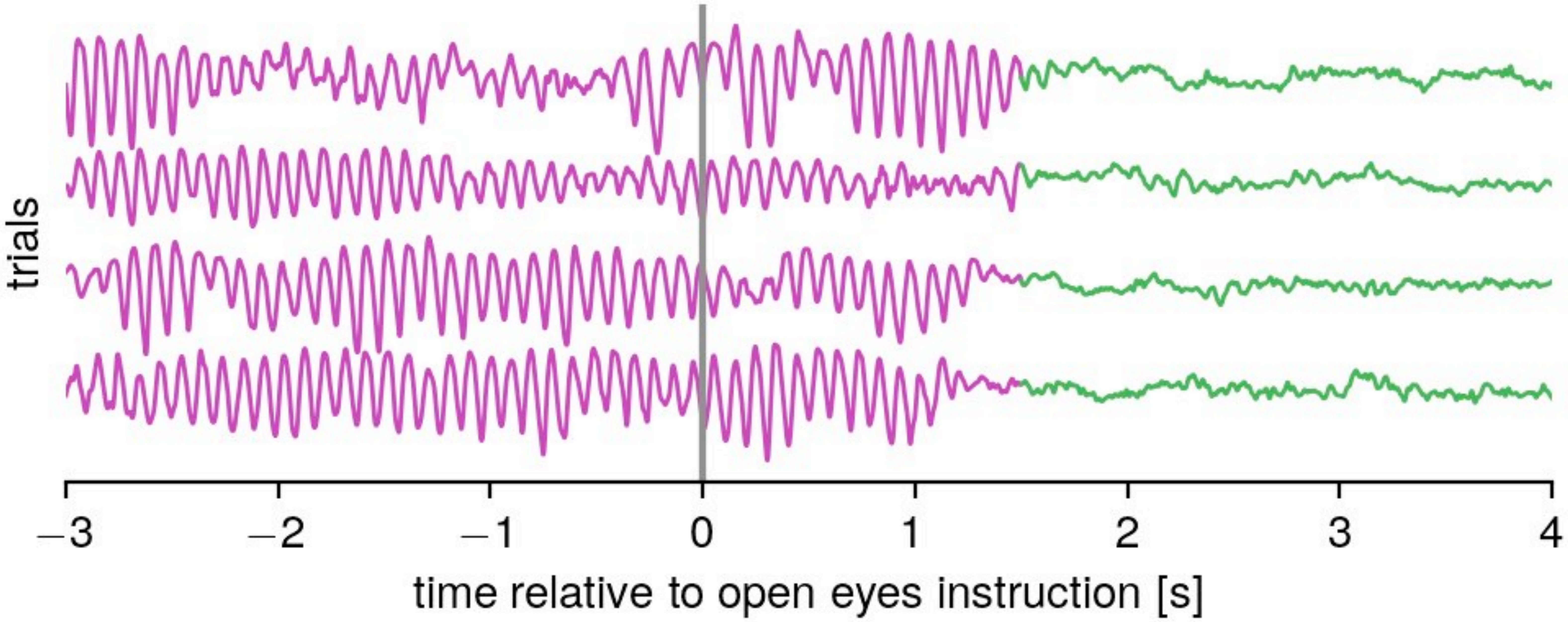
what about phase?

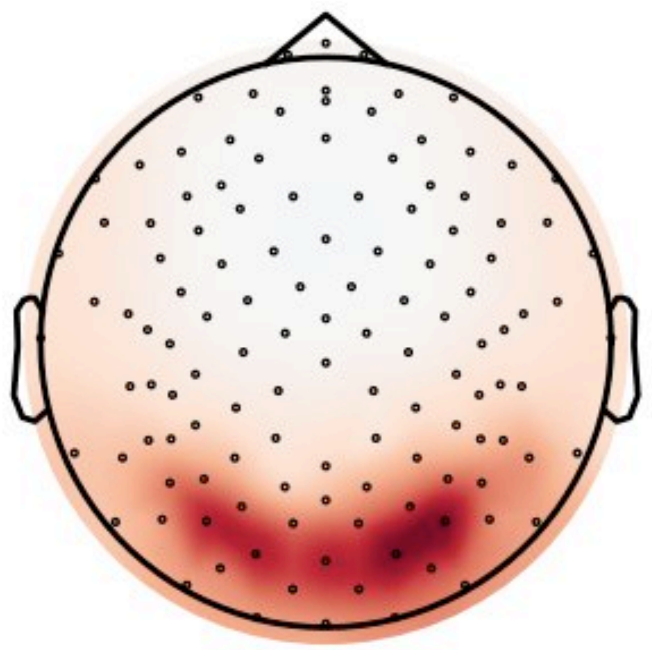


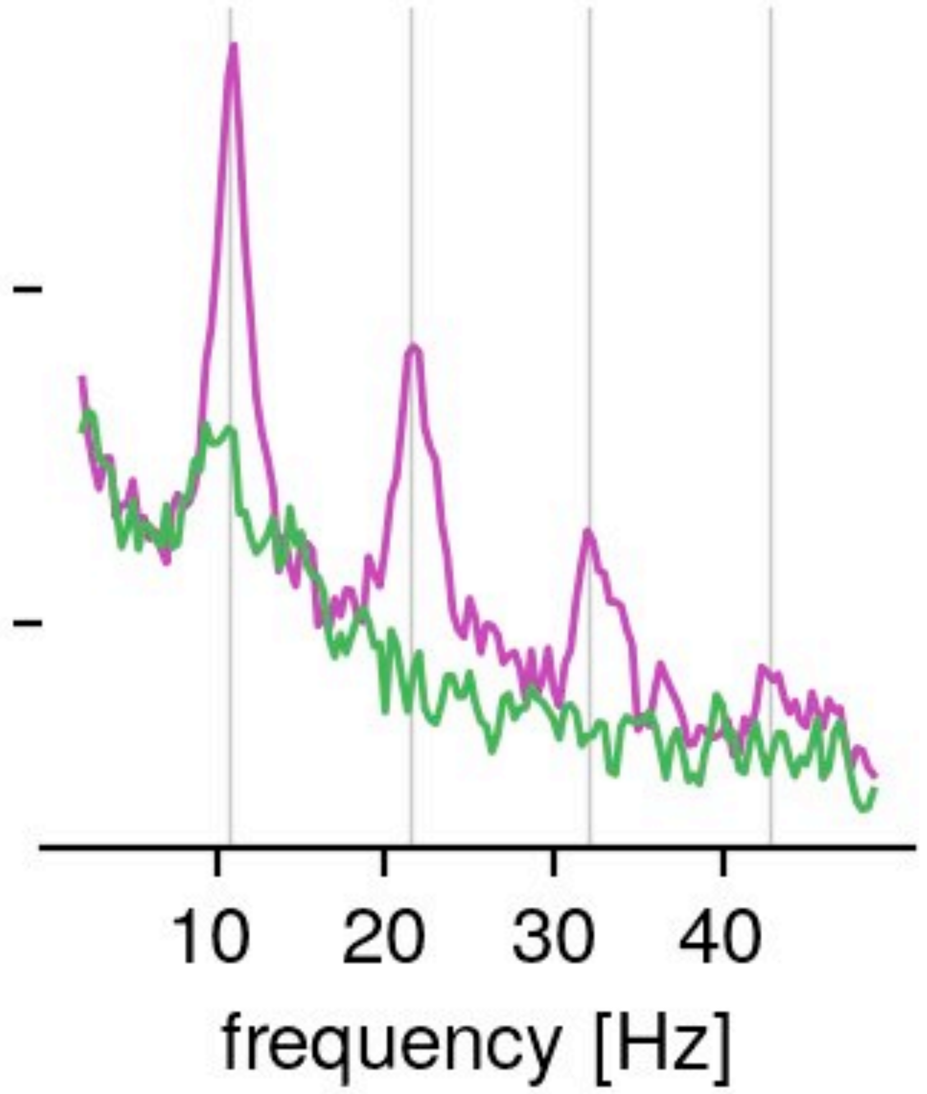
same spectrum – different time domain signals: the power spectrum only captures a part of the available information

time-frequency analysis

motivation





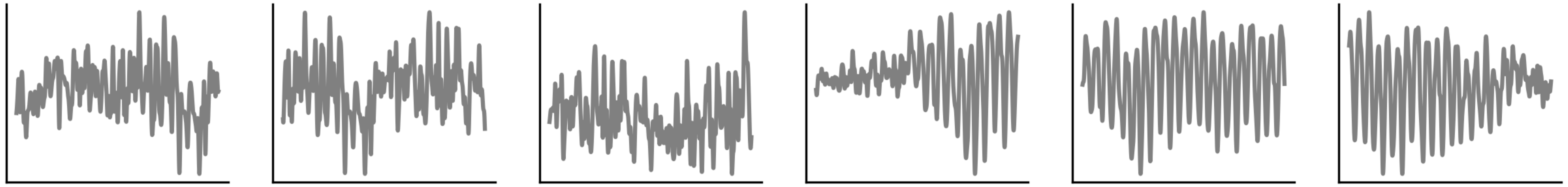


- How to assess temporal dynamics across a trial?

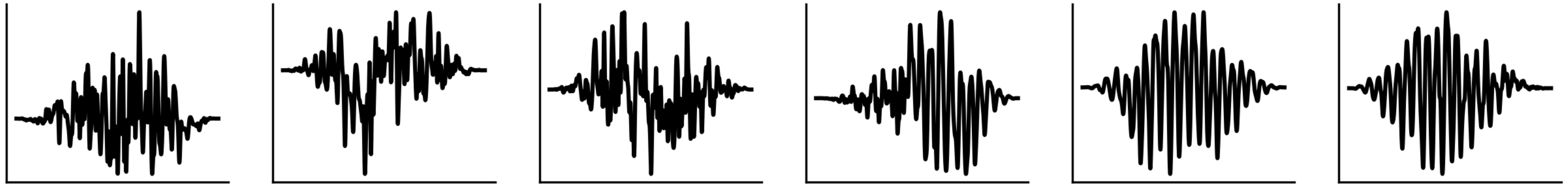
→ time-frequency representation! (TFR)

computation

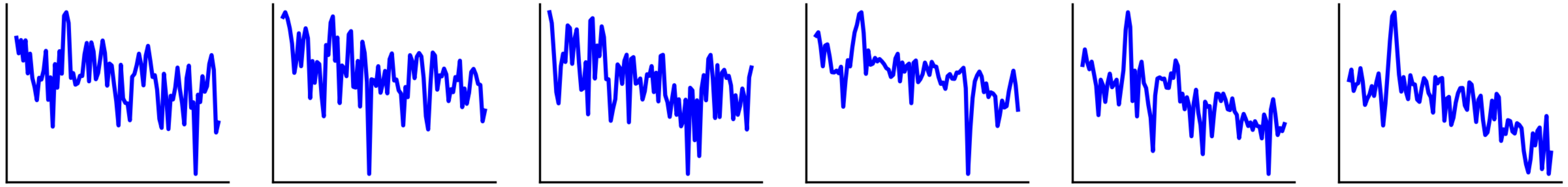
time series, cut into small segments
[nr_segments x nr_samples]



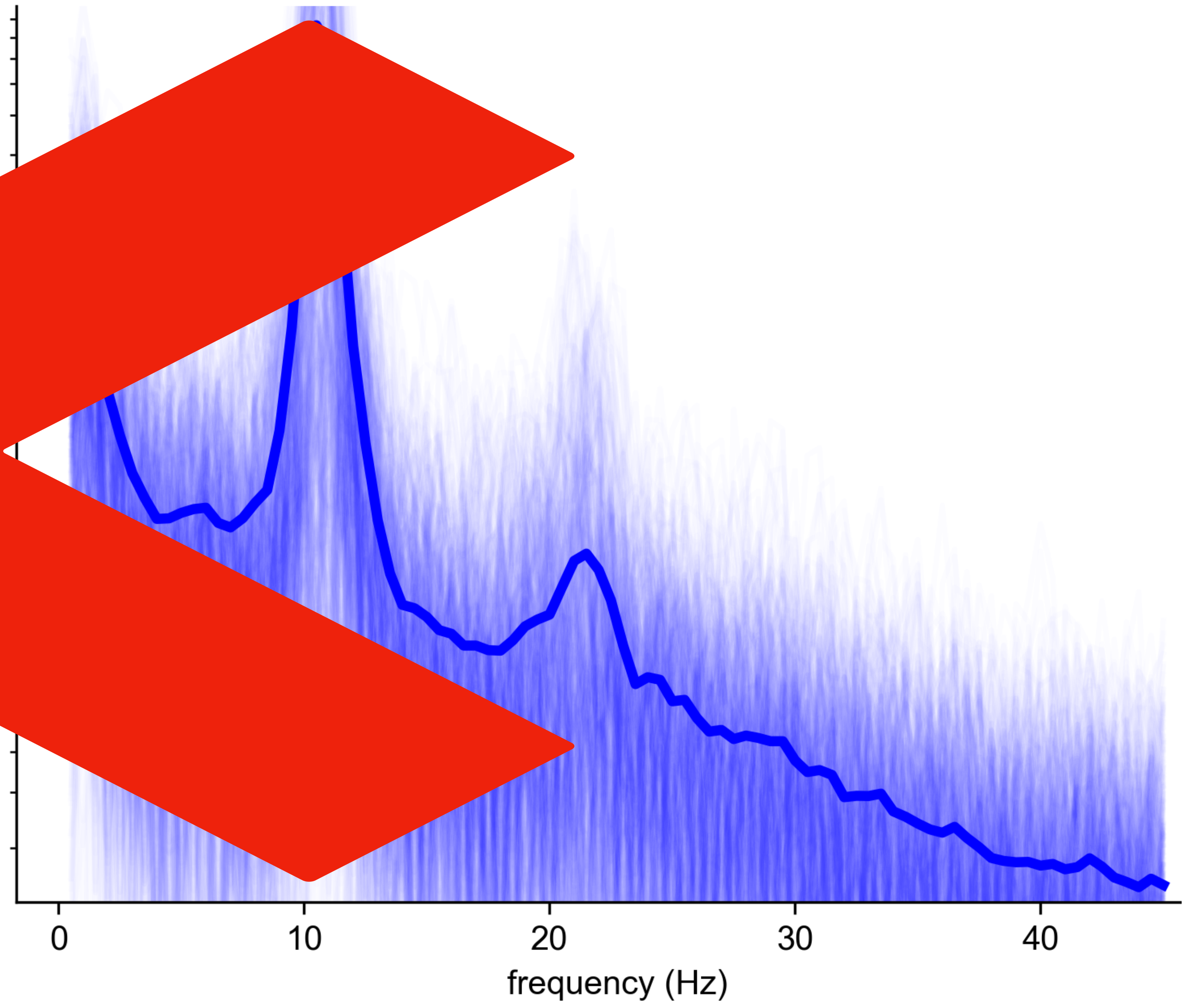
time series, multiplied with a windowing function
[nr_segments x nr_samples]



spectra for each segment
[nr_segments x nr_frequencies]



average across freq
obtain the spectrum
[1 x nr_frequencie

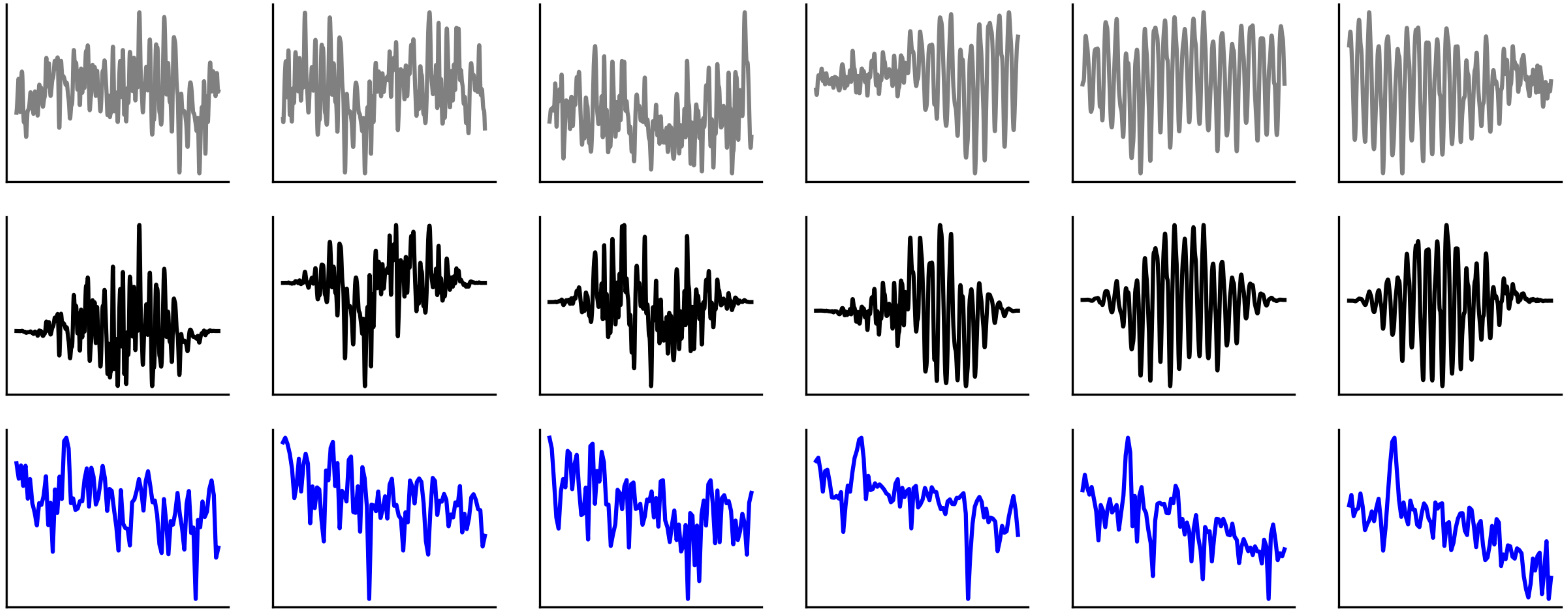


computation

time series, cut into small segments
[nr_segments x nr_samples]

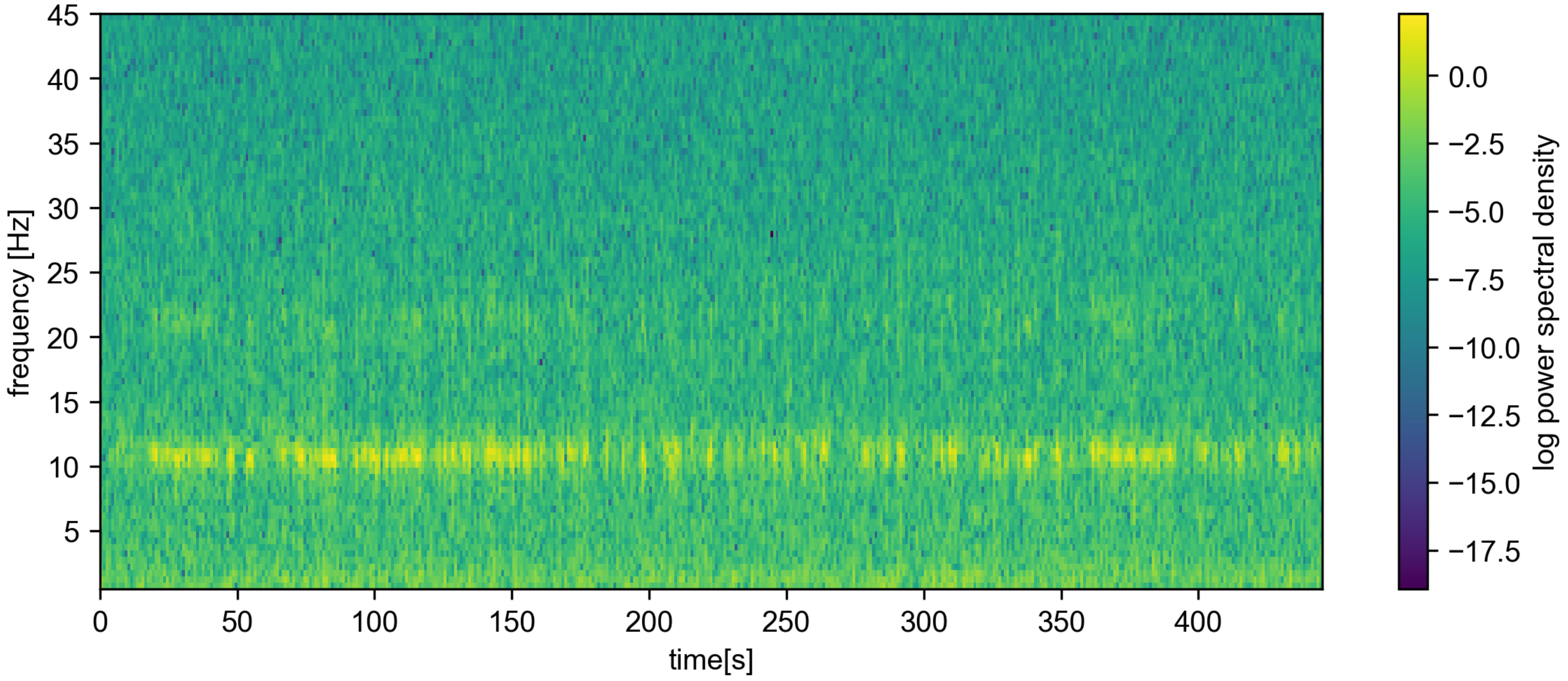
time series, multiplied with a
windowing function
[nr_segments x nr_samples]

spectra for each segment
[nr_segments x nr_frequencies]

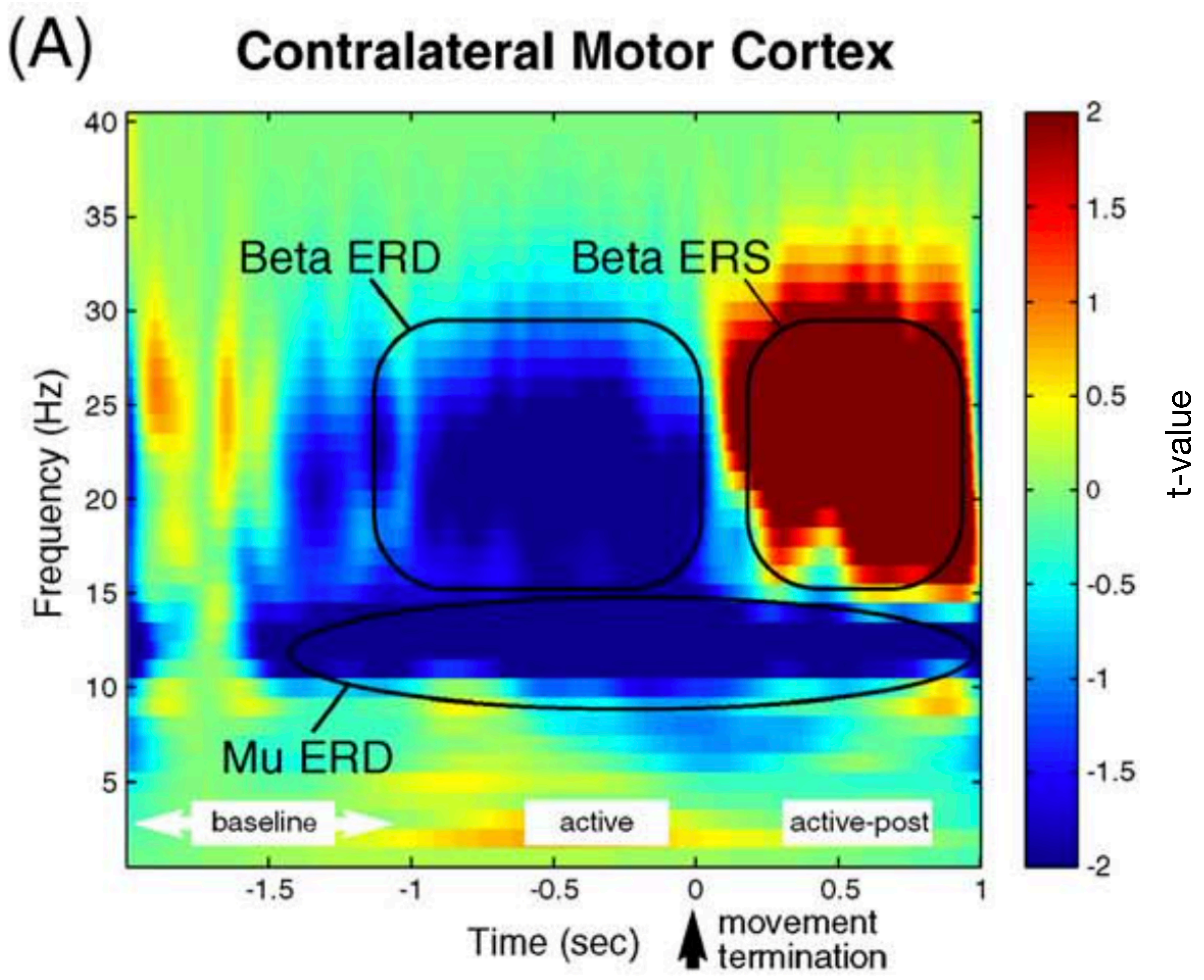


rotate by 90 degrees
stack spectra from segments
& plot in color

→ time-frequency representation



example output for MEG data



- rich spectral power dynamics in different frequency bands
- often, a baseline/ normalization procedure or contrast is employed.

figure from: Jurkiewicz et al. (2006)

methods: parameters

- influence of parameters: window length / number of cycles

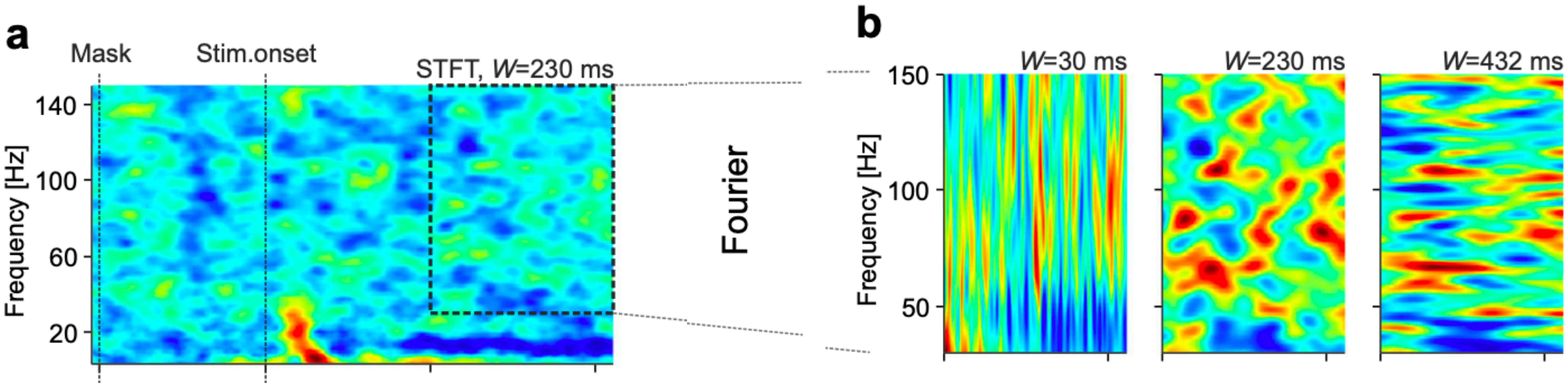


figure from: Moca et al. (2021)

methods: parameters

- different approaches for selecting segment length

fixed window length

no matter the frequency

segment length changes

with frequency

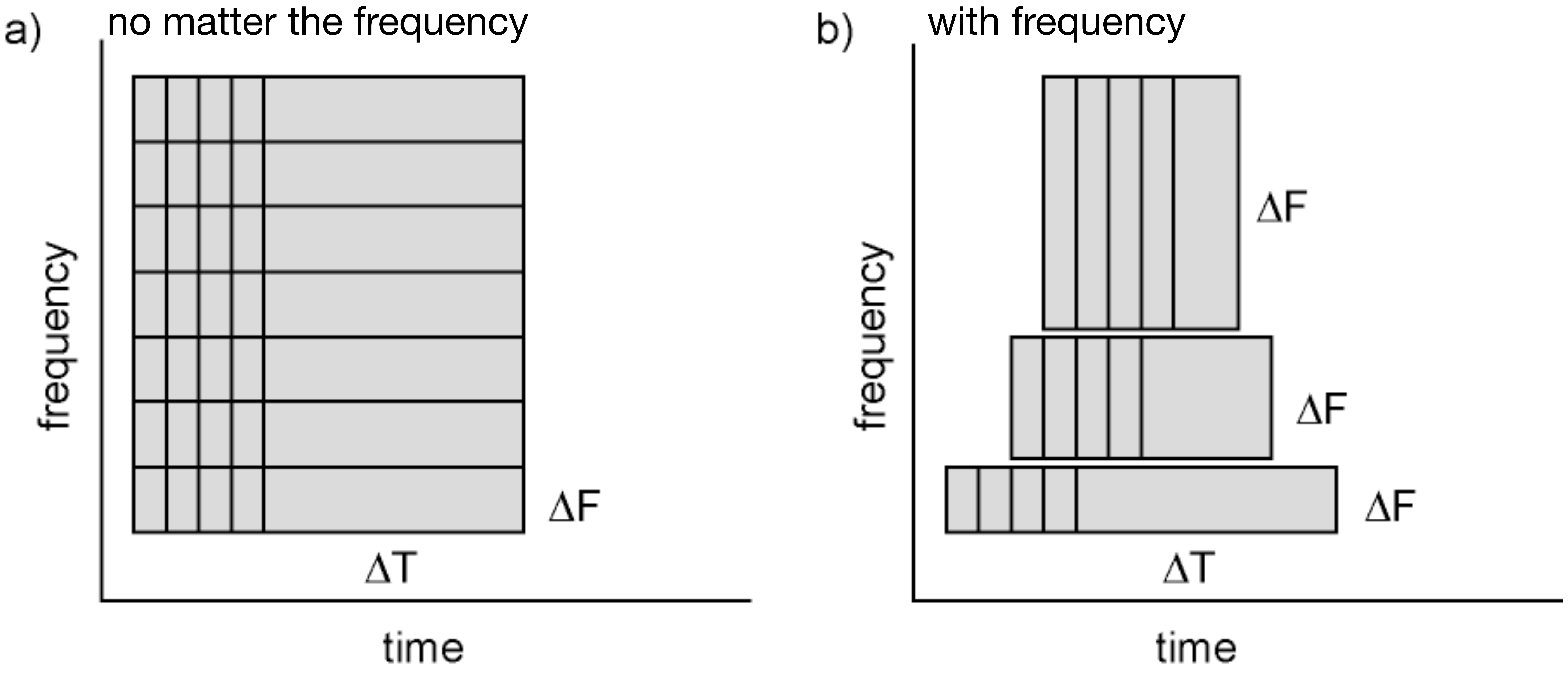
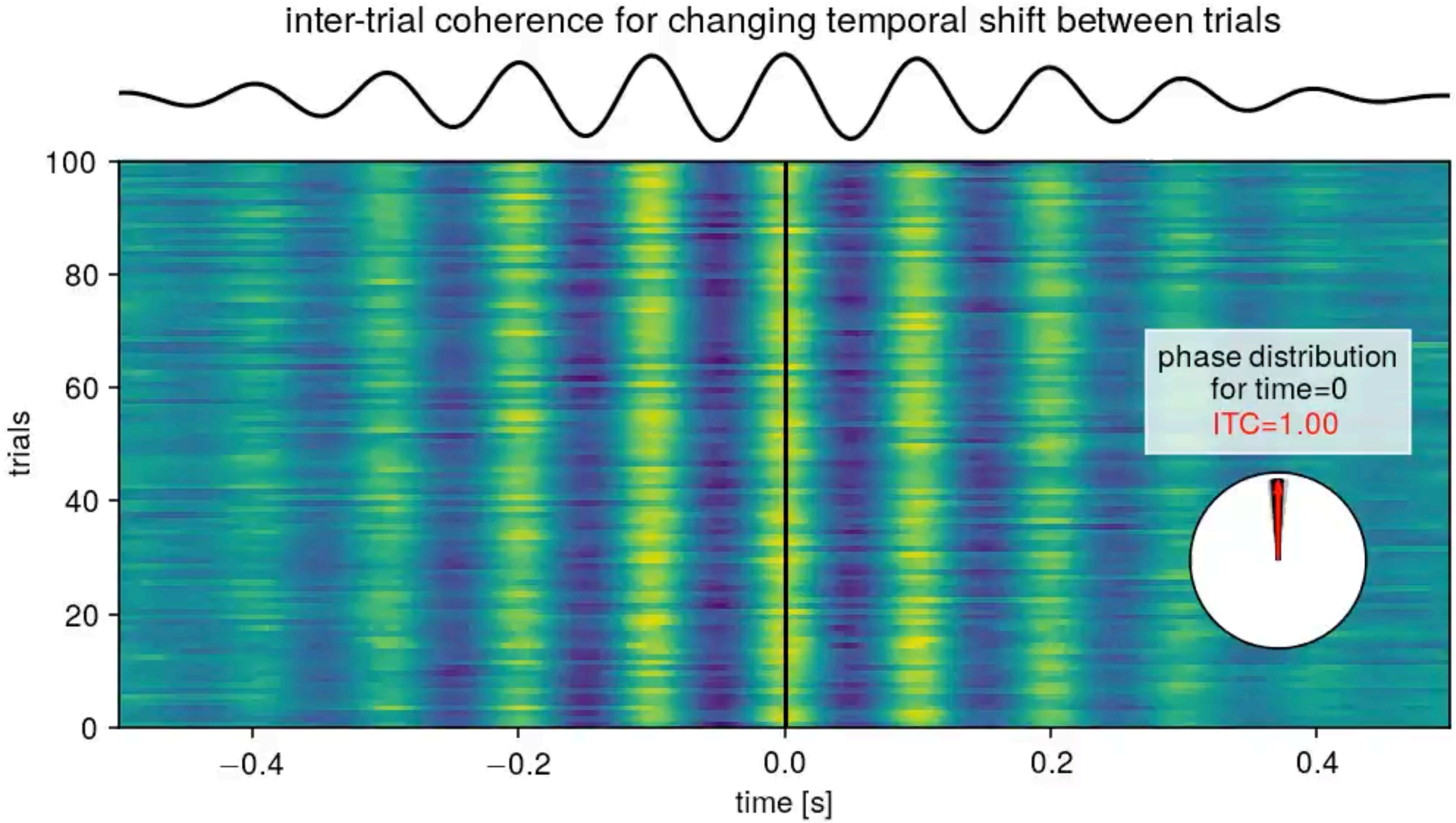


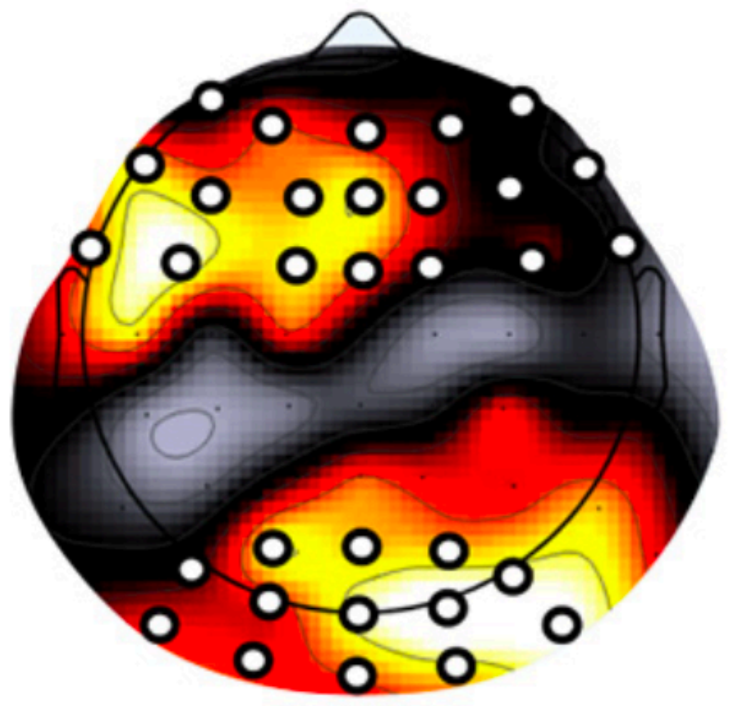
figure from:
https://www.fieldtriptoolbox.org/tutorial/sensor/timefrequencyanalysis

methods – inter-trial coherence

- phase information from the time-frequency representation can be used to investigate inter-trial coherence (ITC)



methods – inter-trial coherence



- comparing ITCs of two different trial groups
- research question: does oscillation phase matter for perception of phosphenes? “optimal phase”
    - two trial groups: phosphenes perceived vs phosphenes not perceived
    - compute ITC for both conditions
    - check if statistically different
- inference: oscillatory phase influences whether participants perceive a phosphene

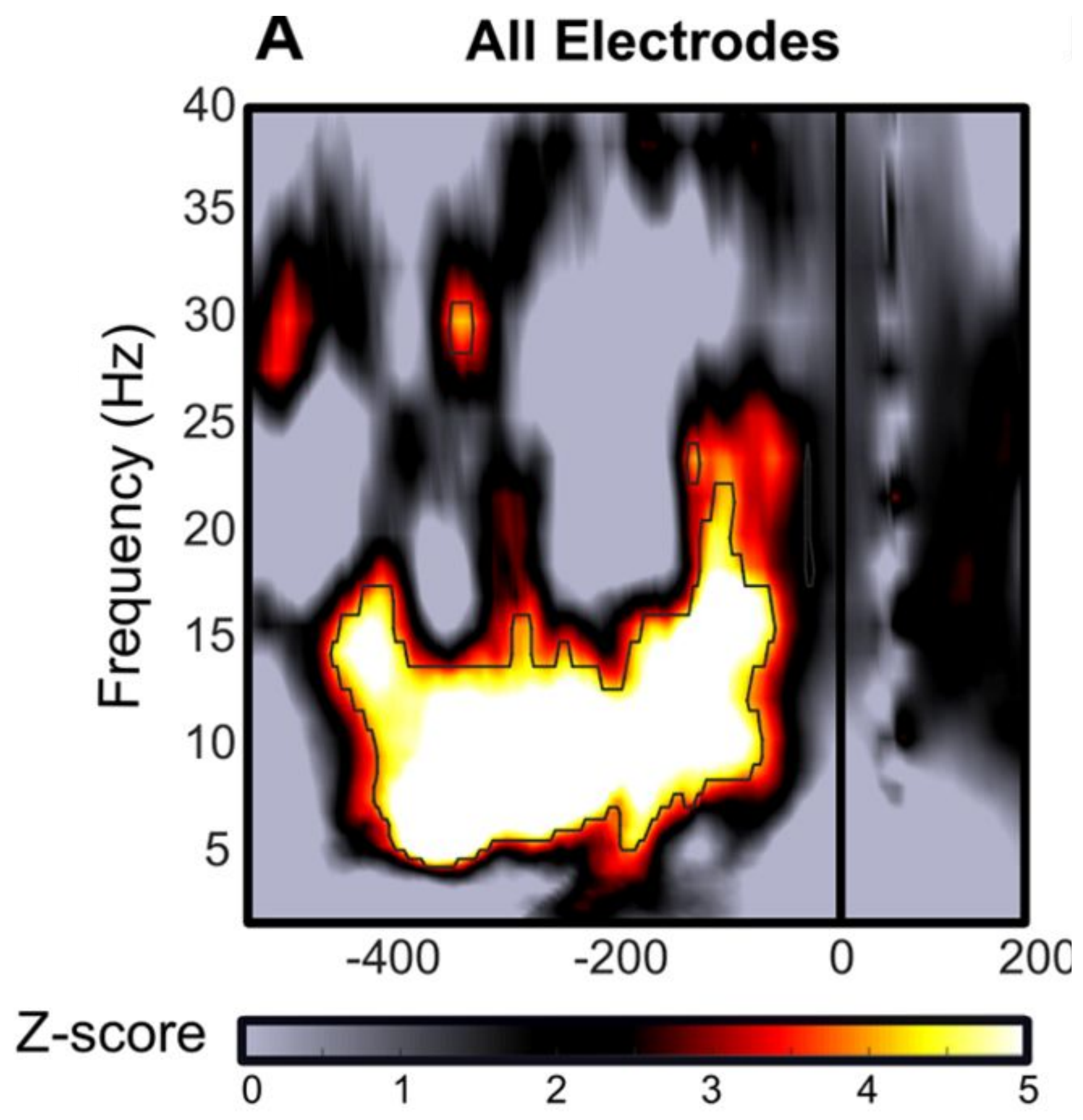


figure from: Fakche et al. (2022)

concepts: evoked vs. induced

- order of operations matters

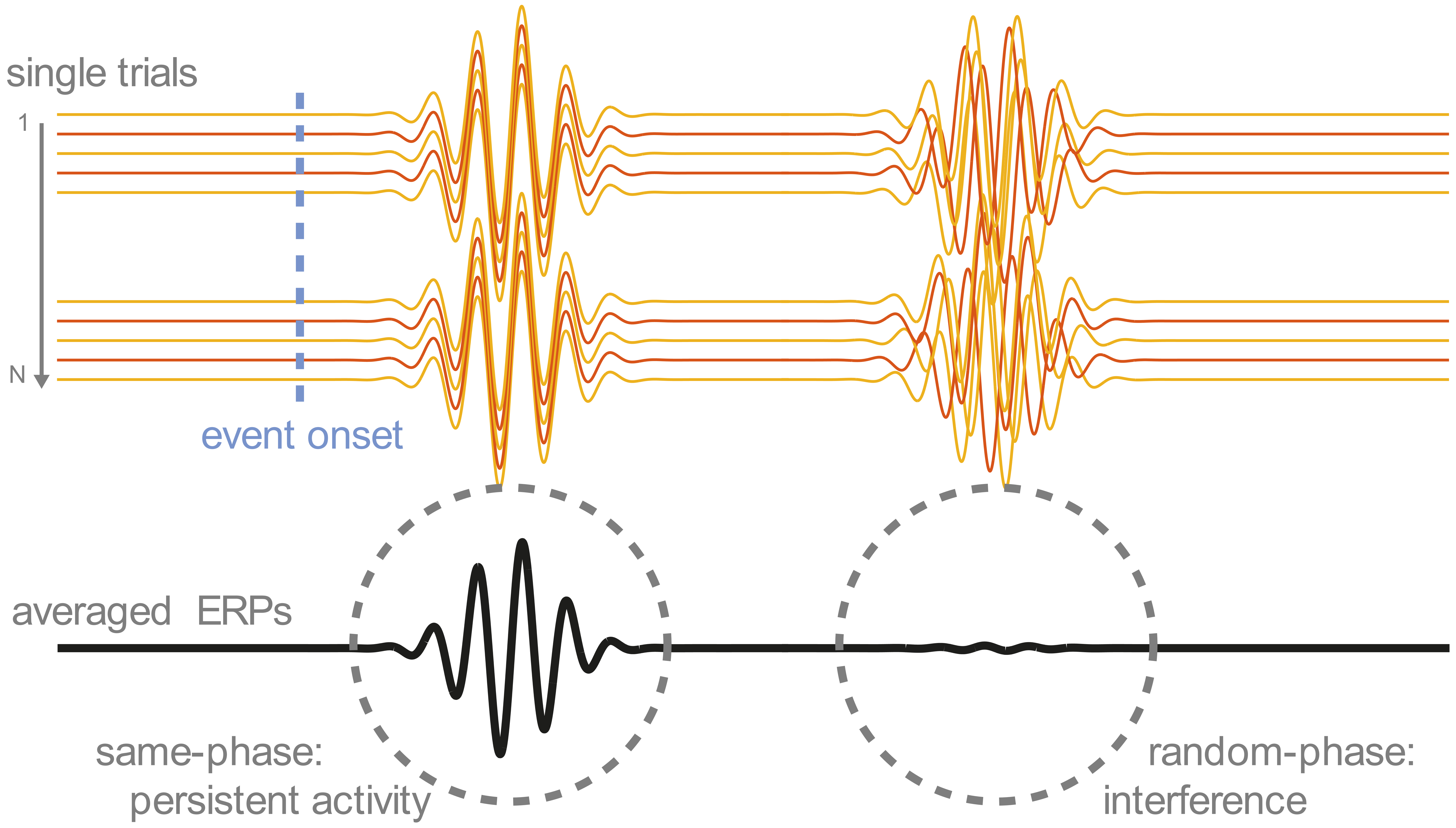
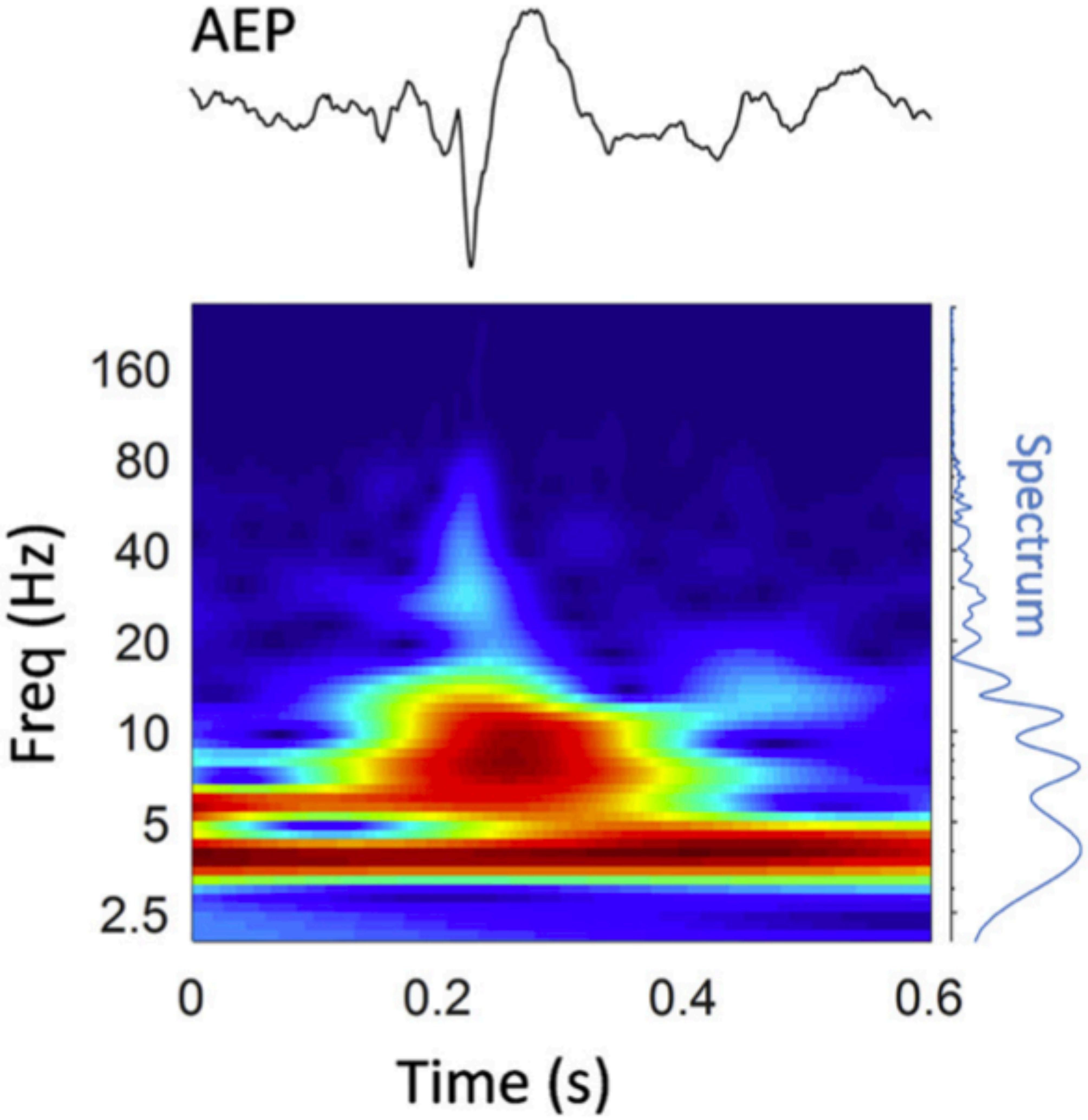


figure from: B. Ehinger:
https://www.s-ccs.de/teaching-resources/open-teaching-graphics/Induced-Evoked/

concepts: oscillations vs. evoked responses



- time-frequency representation of an evoked response shows a peak in the theta frequency band $\Longleftrightarrow$ genuine rhythms

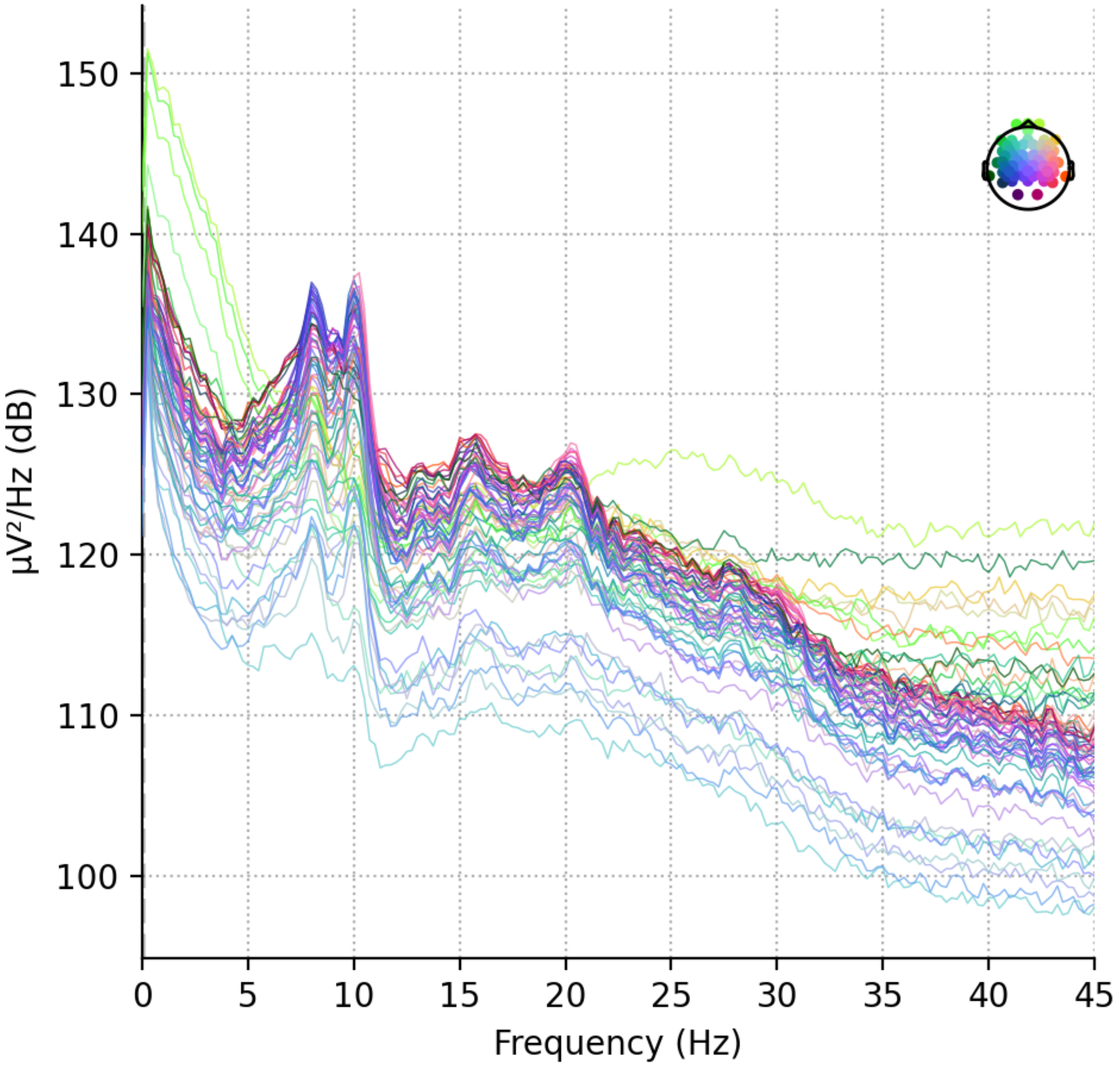
→ TFR on its own is agnostic about generative mechanisms!

figure from: Gourévitch et al., *Neuroscience and Biobehavioral Reviews* (2020)

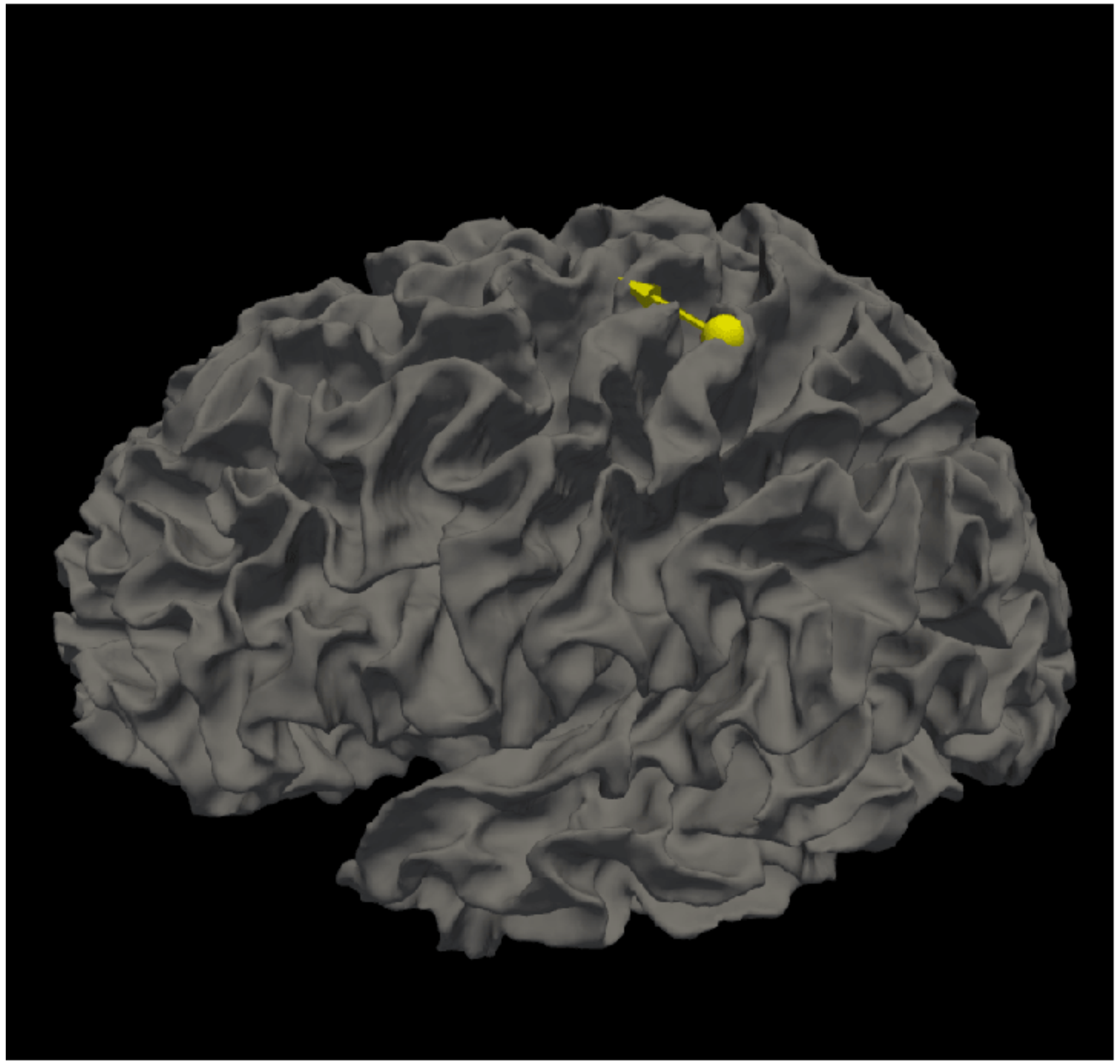
volume conduction

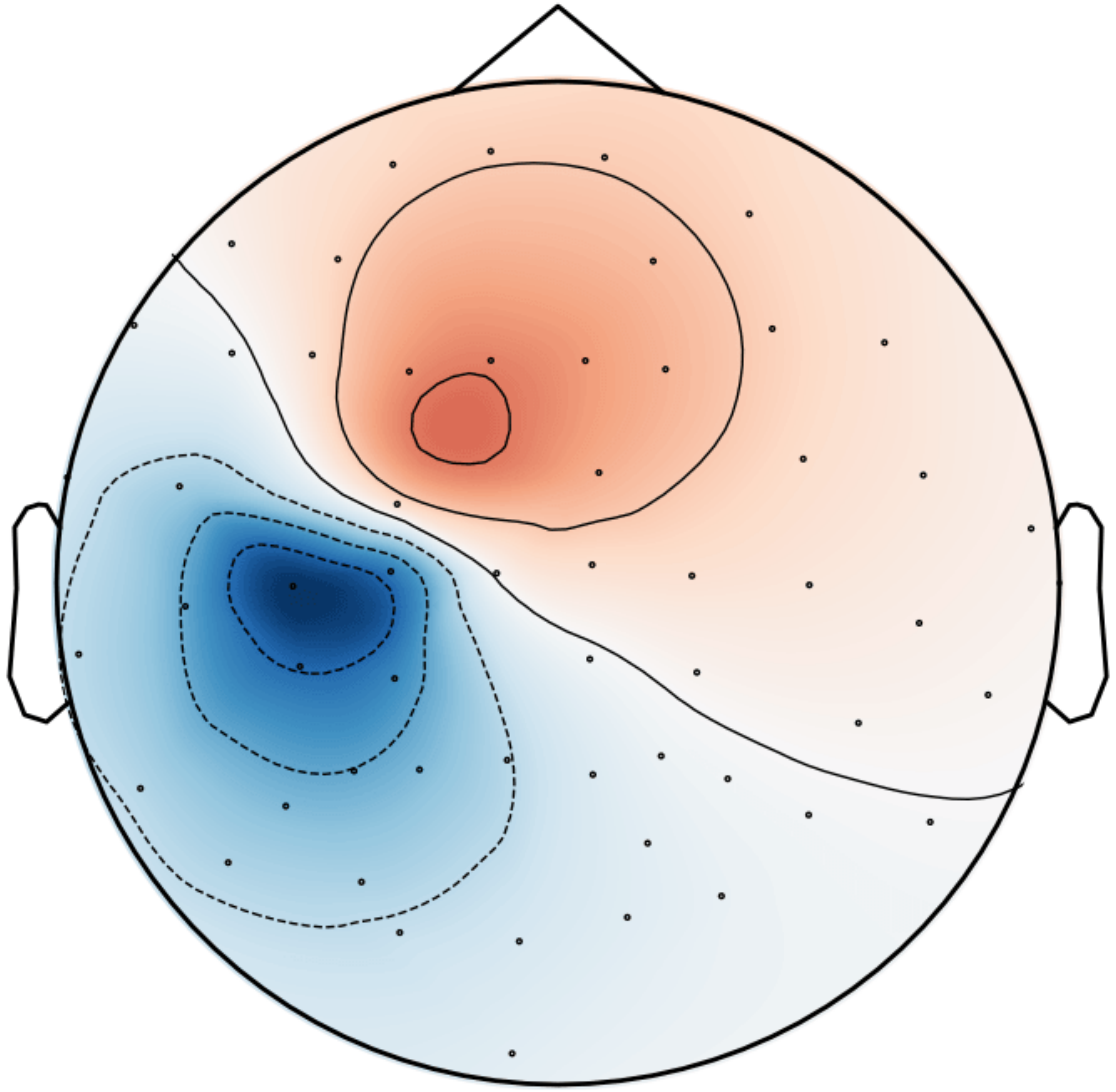
power spectra across channels – data

- channels are not independent
- different signal contributions will overlap and result in multiple spectral peaks

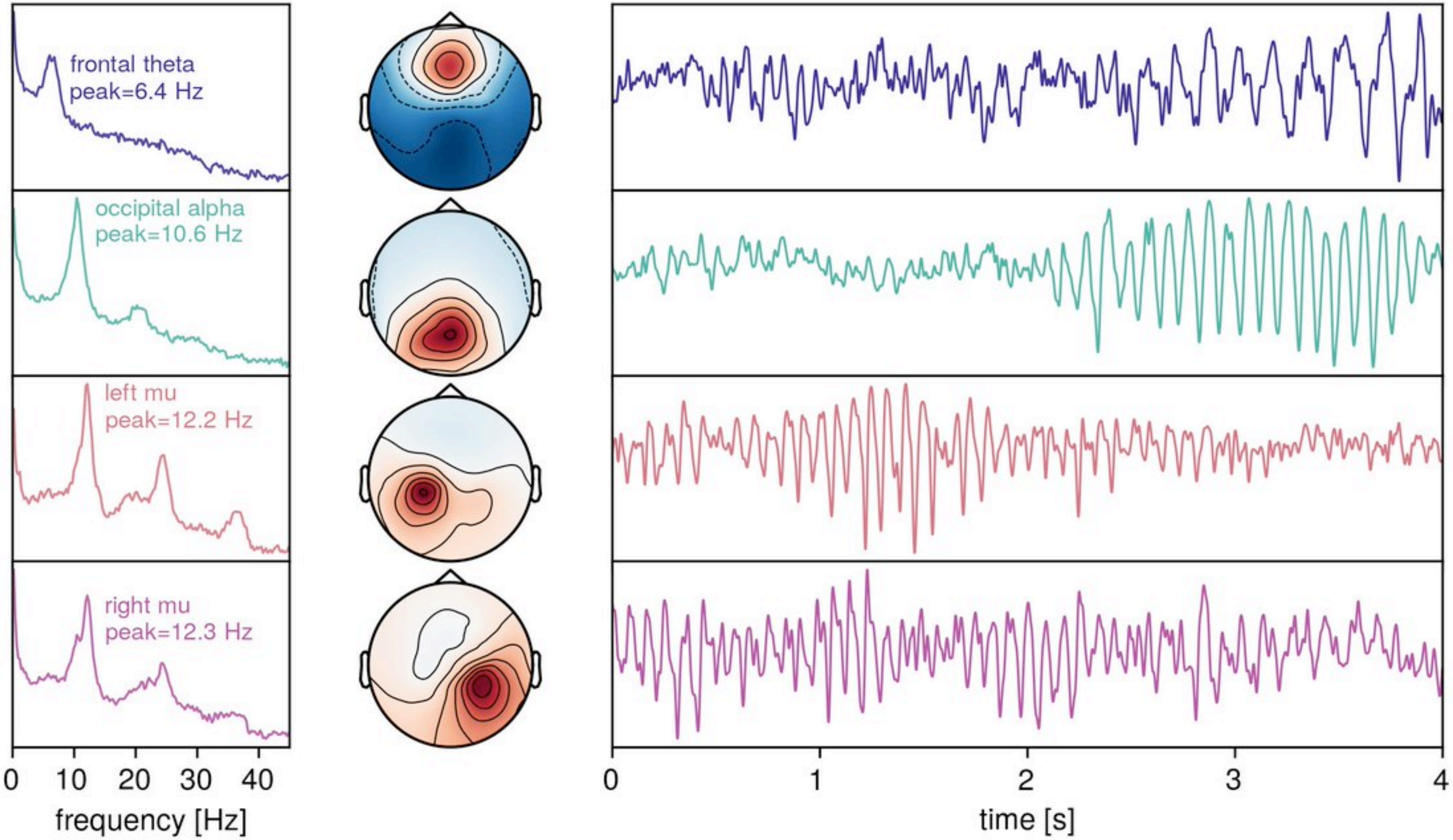


volume conduction – simulation

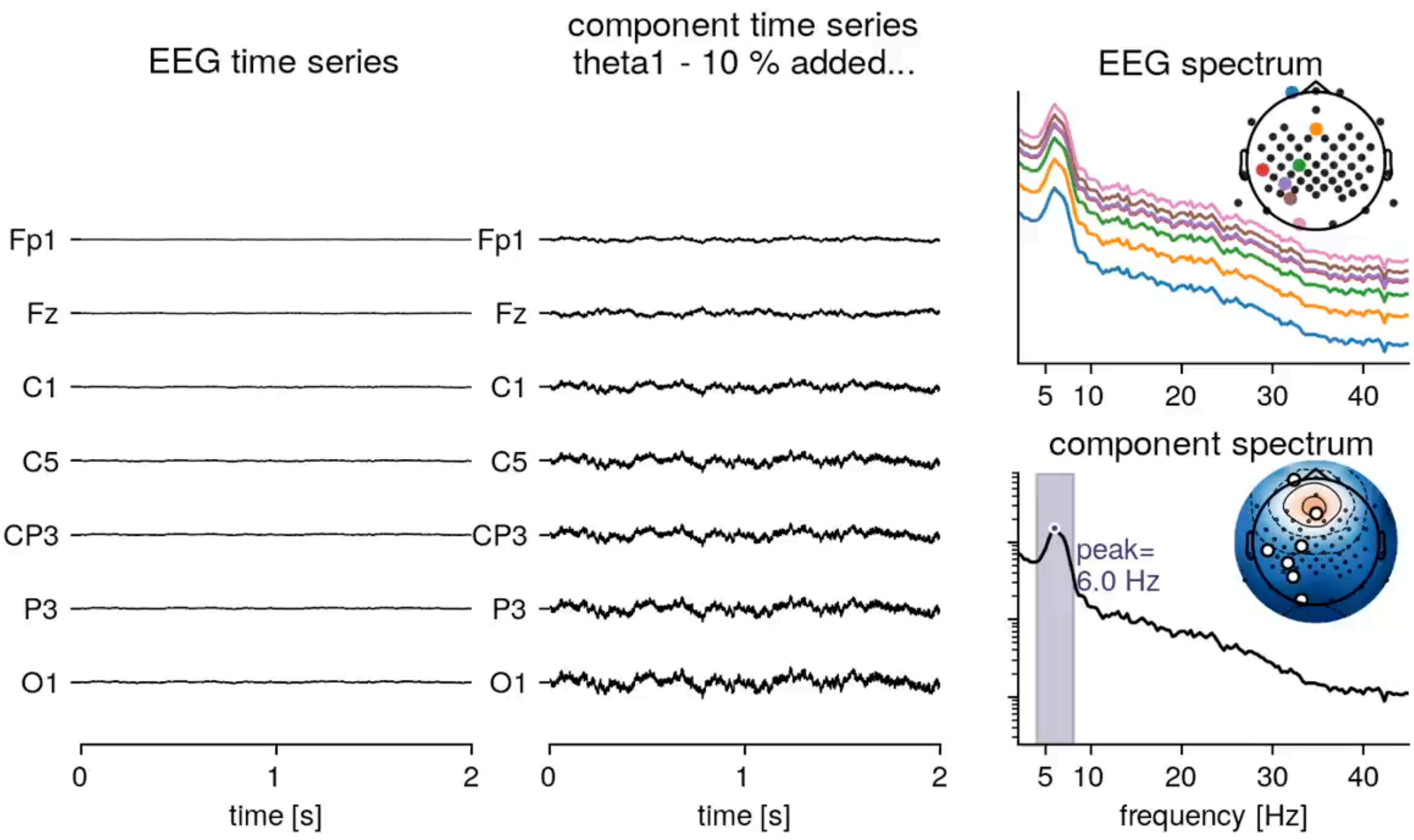




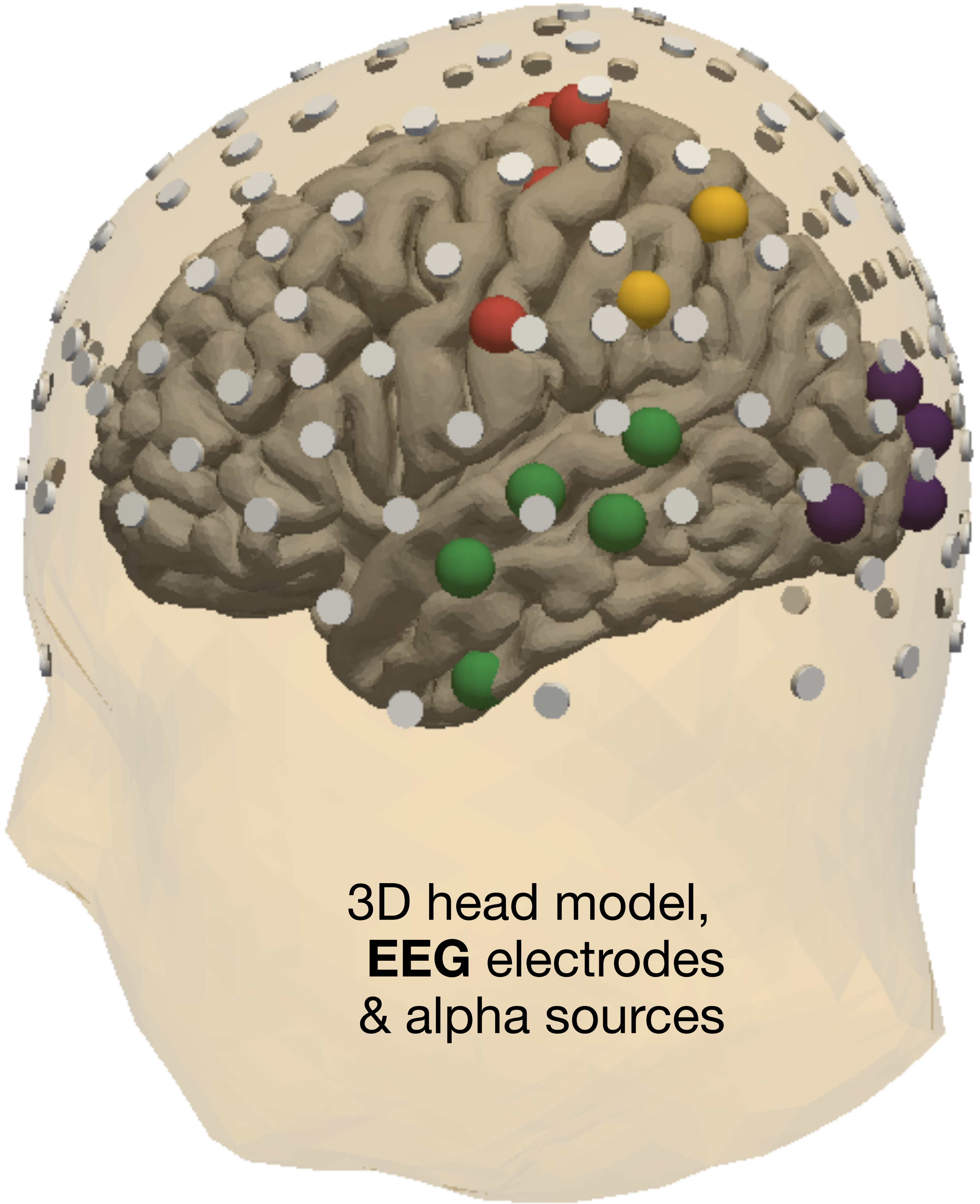
EEG is a mixture of different signals – data

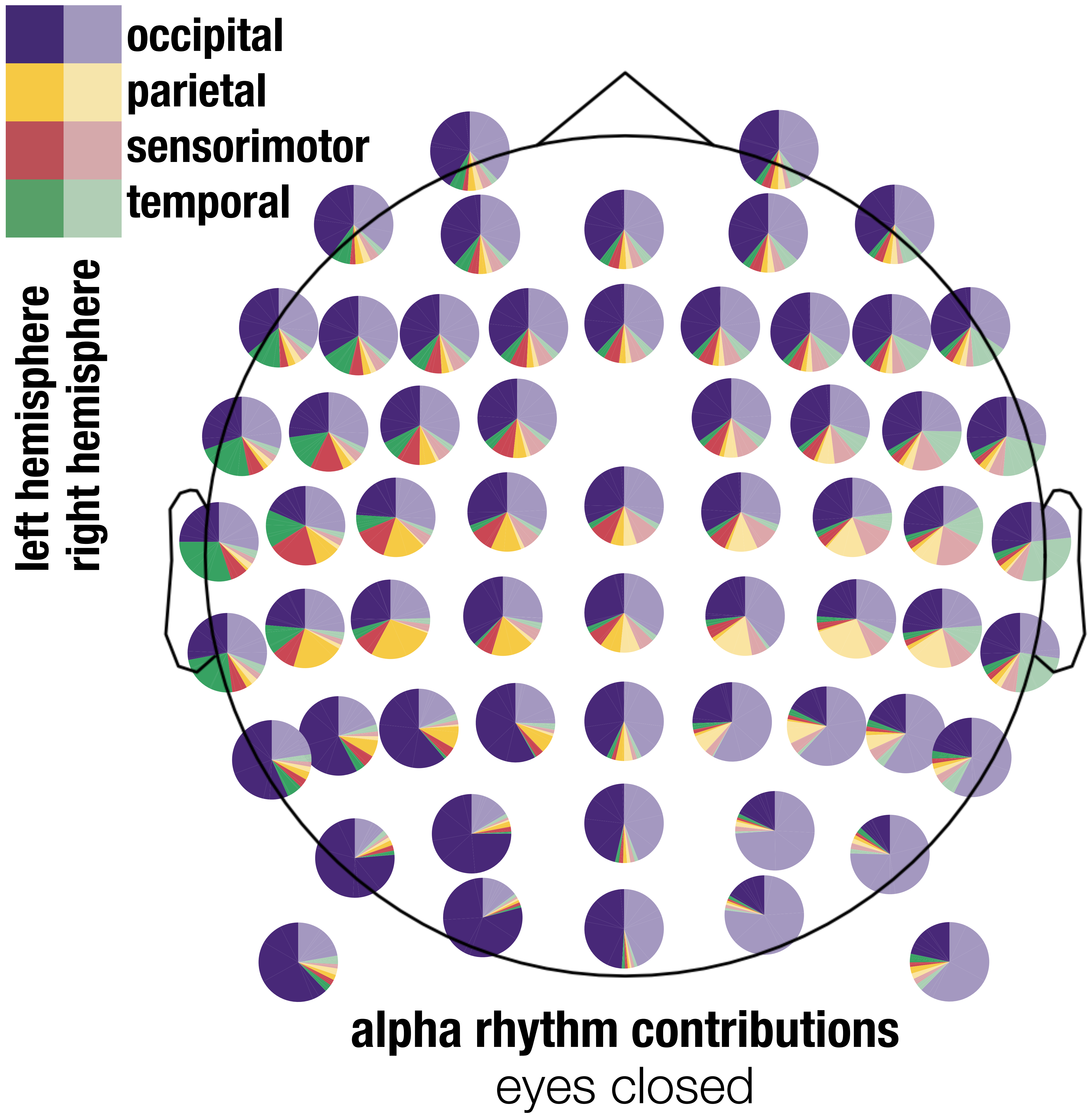


EEG is a mixture of different signals – data



EEG is a mixture of different signals – simulation



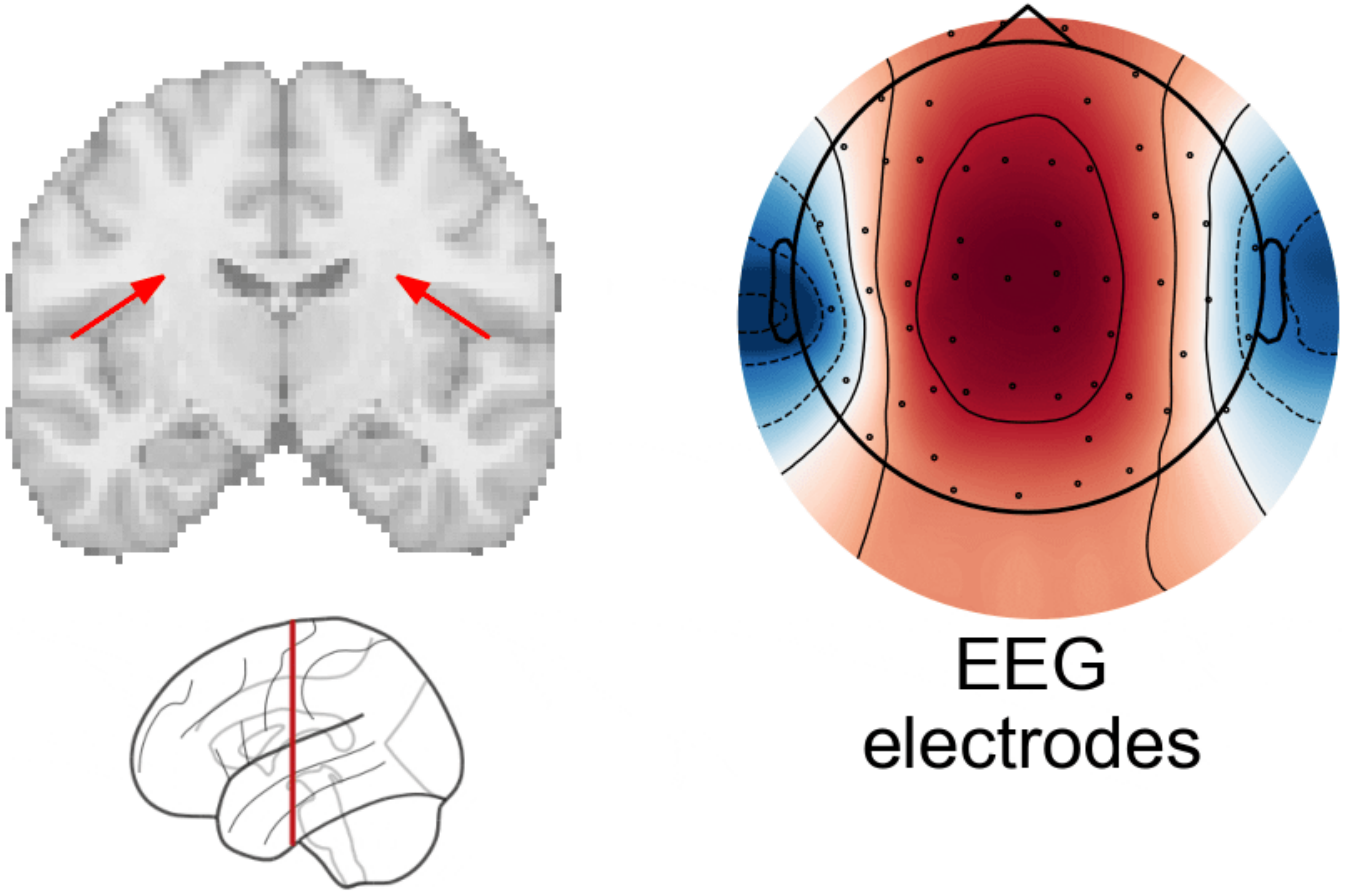


observed spectral peak could come from a region far far away from the electrode.

figure from: Schaworonkow & Nikulin *NeuroImage* (2022)

how do auditory sources end up on Cz? – simulation

- depending on structure + dynamics, activation can show up on sensor locations that are very far away from source locations



tomorrow: source estimation lectures + hands on!


computation
interpretation
Fourier Transform
frequency resolution
multi-taper
Welch's Method
tapers
inter-trial coherence
time-frequency representation
oscillations
amplitude modulation
1/f-activity
harmonics/ waveform shape
generative model of M/EEG
volume conduction
spatial patterns